

MA111: Contemporary mathematics

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SCHEDULE:

- HW 11A due Monday Nov 7, 2011.
- HW 11B due Monday Nov 14, 2011.
- Chapter 11 Exam Monday Nov 14, 2011.

Today we will formalize 2D point groups and look at pretty names.

I expect you to read 11.1-11.6 and exercises 1-48, 55-68, 73-74.

Tomorrow we'll start the material for 11.7, 49-54, 75-80.

11.2 - 11.5: rigid motions

- Chapter 11 studies [symmetries of flat things](#)
- An infinite blank sheet of (flat) paper has four kinds of symmetries:
- **Rotations** These leave one point alone, and swirl the rest around
Specified by the **center** and **angle**
“rotocenter” and “order”
- **Reflections** These leave one line alone, and flip the rest over it
Specified by the **line of reflection**
These switch left with right (or top and bottom)
- **Translations** These move all points, but preserve left-right
Specified by a **direction** and **distance**
“vector”
- **Glide reflections** These move all points, and switch left-right
Specified by a **direction** and **distance**
“vector”

11.1 and 11.6: 2D Point Groups

- If our object is “bounded” then we cannot use translations or glide reflections (it would “float away”)
- The **symmetries of such an object** are only rotations and reflections
- The complete classification:
 - Find the rotation of the largest order, n
 - Does it have reflections?
Yes: D_n
No: Z_n
- D_n has n rotations (including the all-the-way-around 360° does-not-change-anything rotation) and n reflections (all of them really-real actually-flips-some-stuff reflections)
- Z_n has n rotations and 0 reflections

Exercises 57-62: Multiplying symmetries

- “A” and “B” are some symmetries of the flat world
- If “A” leaves a shape alone, and “B” leaves a shape alone, then actually we know some other things that leave the shape alone:
- “A*B” meaning “A, then B” leaves the shape alone
- A rectangle stays the same after a vertical flip, “A” and after a horizontal flip, “B”
- It must stay the same after “A*B” too:
but what is a simplified name for “A*B”?

73-74 and 11.7: Groups

- A **symmetry group** is a list of symmetries so that if “A” and “B” are in the list, so is “A*B” (and so that if “A” is in the list, so is “1/A” the opposite of “A”)
- A symmetry group defines what it means for two points to be the same: if you can get from one to the other using a symmetry in the list
- Symmetry lets us simplify problems (graph theory example)
- A **discrete** symmetry group is one in which there is some distance D such that two points closer than D cannot be the same
- Classification: A discrete symmetry group of the flat world is:
 - Z_n or D_n (the [Rosette Groups](#)), or
 - One of 7 [Frieze Groups](#), or
 - One of 17 [Wallpaper Groups](#)

Project B: More useful symmetry

- In three dimensions there are 6 basic rigid motions
- Rotary reflections and screws are new
- Instead of Z_n and D_n we get 7 letters (and also 7 freakazoids)
- Most of the time these are used for crystal lattices
- Only 32 possible letter/number combinations for crystals
- 230 “full” crystal groups (like Wallpapers)
- There are 80 “almost full” (Layer) crystal groups, and 75 “barely full” (Rod) crystal groups