MA111: Contemporary mathematics

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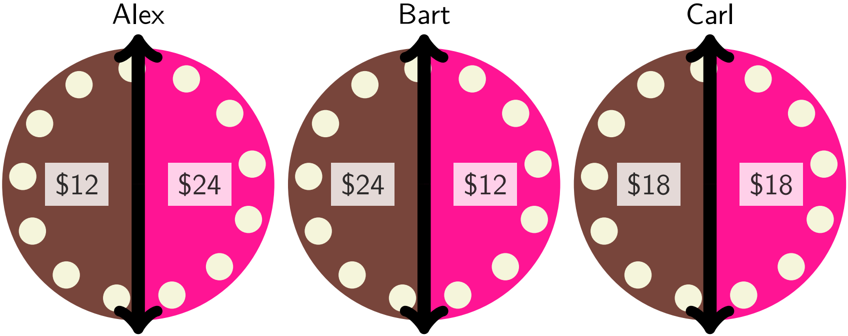
Schedule:

- Homework?

Today we will study how three players may play a game to avoiding fighting.

The loot in the eye of the beholders

- We have a single cake, half-strawberry, half-chocolate, but three people who own 1/3rd of it value it differently:



- We saw last time that the way the cake is cut can cause suffering or it can make it so that everyone is happier than expected!

A game to cut the cake

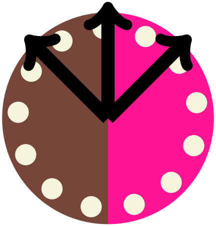
- We saw that maximal total happiness didn't make everyone happy
- We saw that a mind-reading (email-reading) all-powerful altruistic government could make everyone happy
- Can the people find happiness themselves?
- We need a game to cut the cake!
- "Lone divider" is one such game

The rules of Steinhaus's game

- Requirements: three players, a divisible resource
- Rules:
  1. Randomly assign "1st", "2nd", and "3rd" to the players
  2. 1st player divides the resource into three sections
  3. 2nd and 3rd player say "yes" or "no" for each section
     actually they secretly commit to "yes" or "no" without telling the other
  4. If both players said "yes" for one section (the same section),
     4.1 The 2nd player pushes two of the sections (including the yes-section) together, and divides them again
     4.2 The 3rd player says "yes" to one of them
- Outcome:
  - Step 4: Player 1 gets the left-over section not combined in 4.1, Player 3 gets the section they chose in 4.2, and Player 2 gets the rest
  - No step 4: Player 2 and player 3 each get one section they said yes to, Player 1 gets the rest

Strategy for player 1: One good piece for me

- Suppose Alex is player 1, and divides the cake like this:



- What do you think Bart and Carl will do?

Strategy for player 1: One good piece for me

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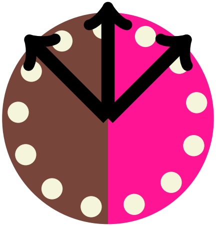

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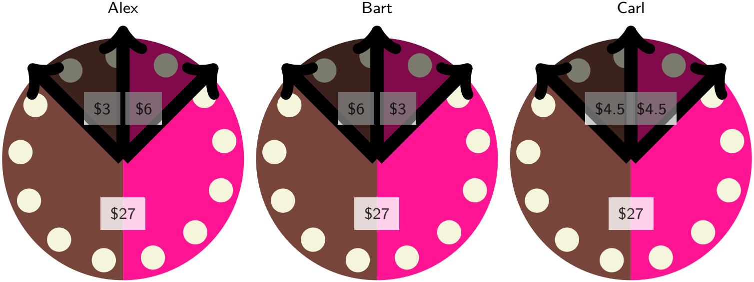
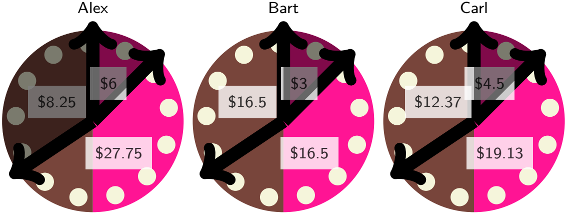


One good piece for me: how does it go?

- So Bart gets to recombine two of the pieces.

Let's assume he uses the winning strategy:



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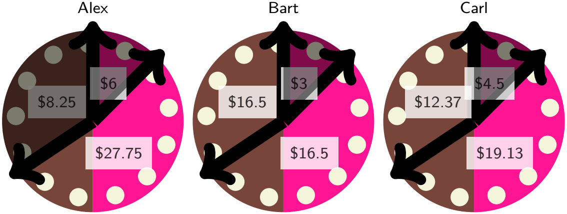
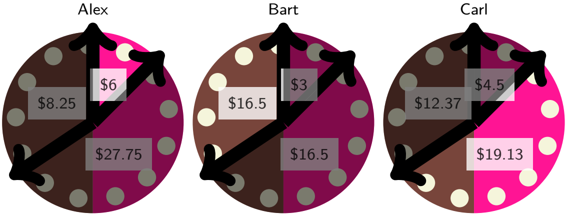


- Carl's strategy seems clear: Choose the good piece

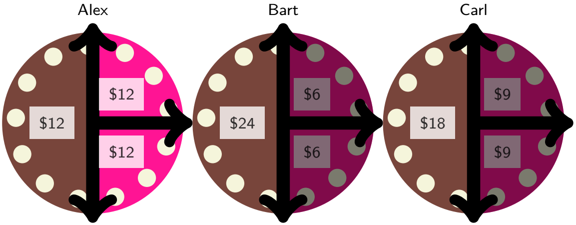


Good piece for me: final results

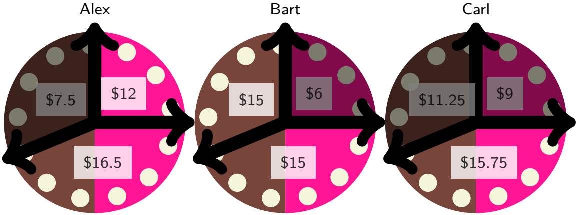
- Alex: \$6 of cake for \$12 paid, ☹
- Bart: \$16.50 of cake for \$12 paid, ☺
- Carl: \$19.13 of cake for \$12 paid, ☺
- Total: \$41.63 of cake for \$36 paid, ☺?
- What did Alex do wrong?
  He made a section he didn't want (and then got it!)

Winning strategy for Player 1: Honesty

- How should Alex avoid getting a piece he doesn't want?
- He should divide the cake fairly!

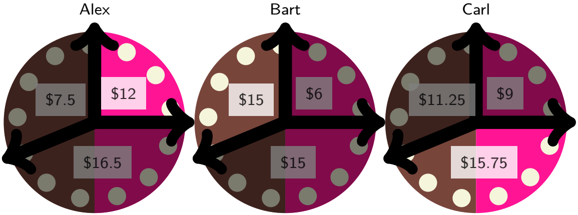


- Bart and Carl both only want the Chocolate half, so Bart divides it:



Winning strategy: Results

- Alex gets the unclaimed, Carl chooses, then Bart gets the rest



- Alex: $12 for $12, ☺
- Bart: $15 for $12, ☺
- Carl: $15.75 for $12, ☺
- Total: $42.75 for $36, ☺

Who goes first?

- Alex only got \$12 for \$12, while the other two did better.
- 1st player's winning strategy always results in an exactly fair share.
- 2nd player's winning strategy does a little better, but
- 3rd player always gets his favorite share
- We have to randomize the order to keep it fair

Harder questions

- Does the winning strategy always win? What if Bart and Carl are sociopaths, or at least HATE Alex. Can Alex still guarantee he gets 1/3?

- Does the 1st player have a better strategy?

  he already has a winning strategy, so he doesn't need better, but maybe he is greedy

- Can we make the game give equitable shares, so each player gets a piece of the "extra"?

- What happens with 4 players?