MA162: Finite mathematics

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University of Kentucky

September 28, 2011

Schedule:

- HW 3.1 is due Friday, Sep 30th, 2011.
- HW 3.2-3.3 are due Friday, Oct 7th, 2011.
- HW 4.1-4.2 are due Friday, Oct 14th, 2011.
- Exam 2 is Monday, Oct 17th, 5:00pm-7:00pm in CB106.

Today we will cover 3.1: graphing inequalities

Exam 2: Overview

- 50% Ch. 3, Linear optimization with 2 variables

  1. Graphing linear inequalities

  2. Setting up linear programming problems

  3. Method of corners to find optimum values of linear objectives

- 50% Ch. 4, Linear optimization with millions of variables

  1. Slack variables give us flexibility in RREF

  2. Some RREFs are better (business decisions) than others

  3. Simplex algorithm to find the best one using row ops

  4. Accountants and entrepreneurs are two sides of the same coin

- Mr. Marjoram decides to use his inventory to make that money

- Each of his products earns him some profit

	Panda	Dog	Bird	Inventory
Plush	1.5	2	2.5	110
Stuff	30	35	25	1400
Trim	12	8	5	350
Profit	10	15	12	0

- How many of each product should he make in order to maximize profit only using the materials he already has?

- Work on it in groups. Best group answer (size up to 5) gets an extra point on exam 1.

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- How many of each product should he make in order to maximize profit only using the materials he already has?

- Work on it in groups. Best group answer (size up to 5) gets an extra point on exam 1. Can you beat $636?

3.1: Inequalities

- Xylophones cost \$200 each and Yukuleles cost \$100 each

- Your need instruments for your new band Glük-N-Spiel

- Your insane and rich uncle only gave you a budget of \$1000
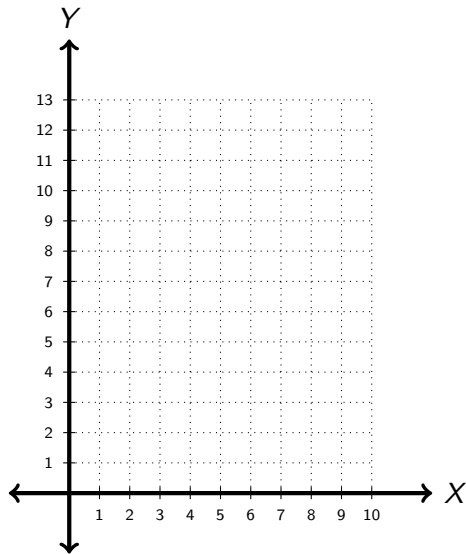
- What are your options?

$$200x + 100y = 1000$$

3.1: Inequalities

- Xylophones cost $200 each and Yukuleles cost $100 each

- Your need instruments for your new band Glük-N-Spiel

- Your insane and rich uncle only gave you a budget of $1000

- What are your options? Don't have to spend it all!
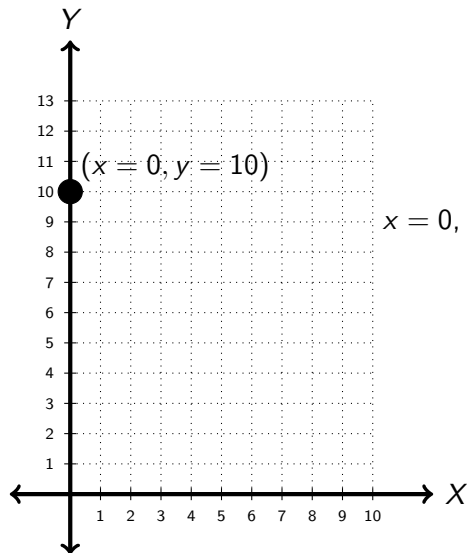
$$200x + 100y \leq 1000$$

3.1: Graphing inequalities

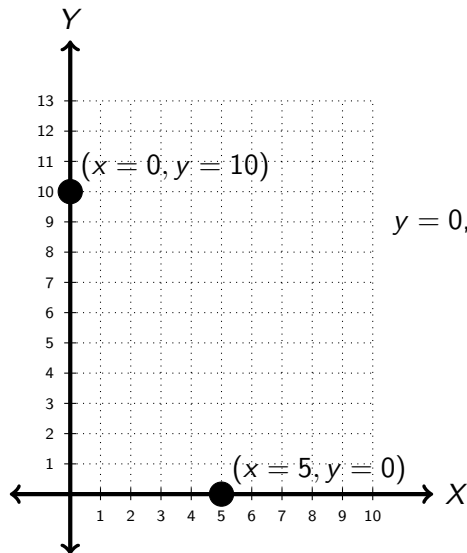$$200x + 100y = 1000$$

3.1: Graphing inequalities

$200x + 100y = 1000$
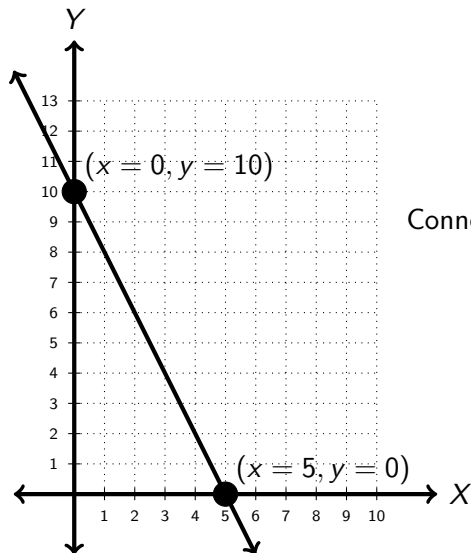
$x = 0,\ 100y = 1000,\ y = 10$

$(x = 0, y = 10)$

3.1: Graphing inequalities

$$200x + 100y = 1000$$

$$y = 0, \ 200x = 1000, \ x = 5$$

Points shown on graph: $(x = 0, y = 10)$ and $(x = 5, y = 0)$

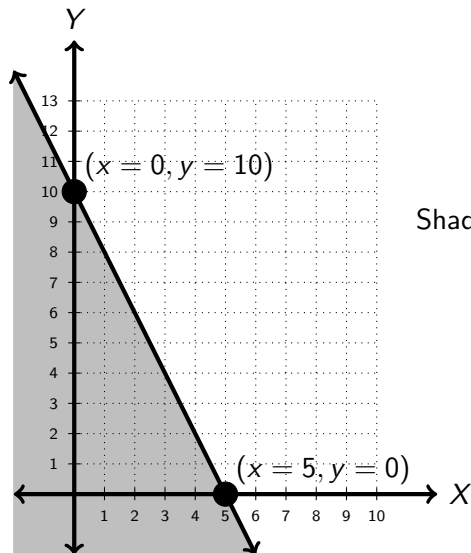## 3.1: Graphing inequalities

$$200x + 100y = 1000$$

Connect the dots

3.1: Graphing inequalities

$200x + 100y \leq 1000$

Shade the region

3.1: Graphing inequalities

- First graph the "equality", that is, graph the line

$\Rightarrow$ Find two points on the line and then draw the connection

- Next graph the inequality, that is, shade the region

$\Rightarrow$ Choose a point not on the lines and see if it is on the correct side

- For example (0,0) is on the correct side since

$$(200)(0) + (100)(0) \leq 1000$$

3.1: Is it realistic?

- Our region is very large.

- Some points don't make sense for a single purchaser:

$\Rightarrow$ $(2.5, 3.5)$ means buy 2.5 Xylophones and 3.5 Yukuleles (\$850)

- But maybe it makes sense as an average or a strategy

- Some points don't make any sense for any purchaser:

$\Rightarrow$ $(-10, -20)$ means buy -10 Xylophones . . . (-\$4000)
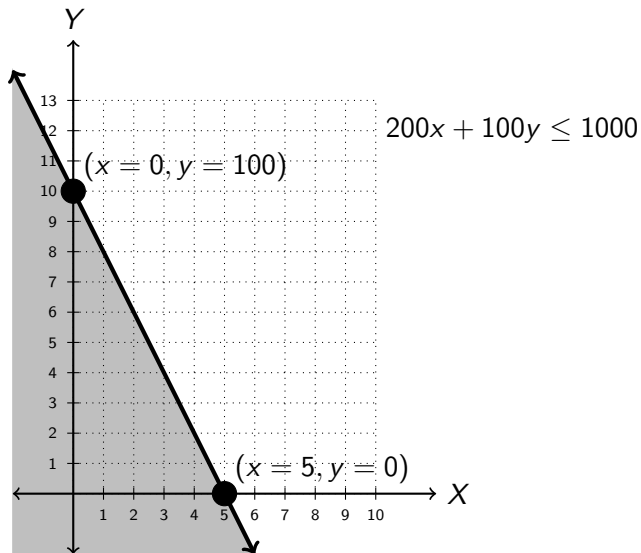
3.1: Systems of inequalities

- We also need some sanity: $X \geq 0$ and $Y \geq 0$

- So we have a system of inequalities:

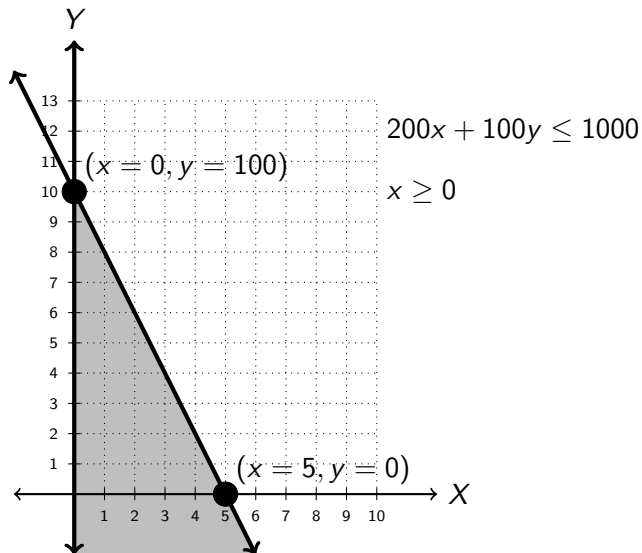$$\begin{cases} 200X + 100Y \leq 1000 \\ X \geq 0, Y \geq 0 \end{cases}$$

- Not enough for just one to be true!

$\Rightarrow$ $(500, 0)$ would be very expensive (\$100,000) and noisy!

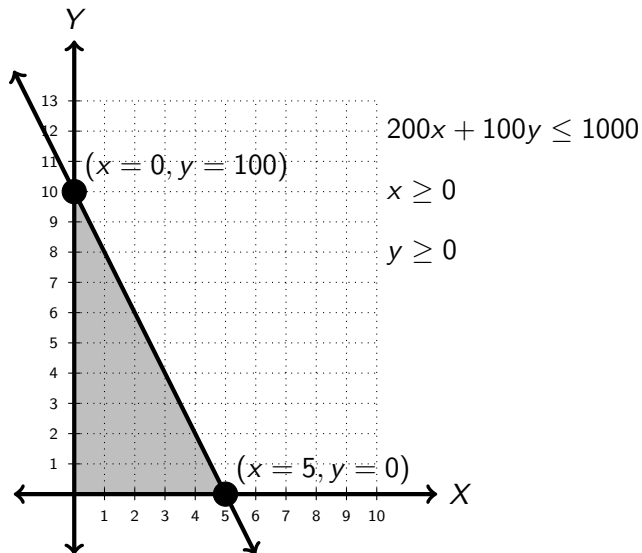## 3.1: Graphing systems of inequalities
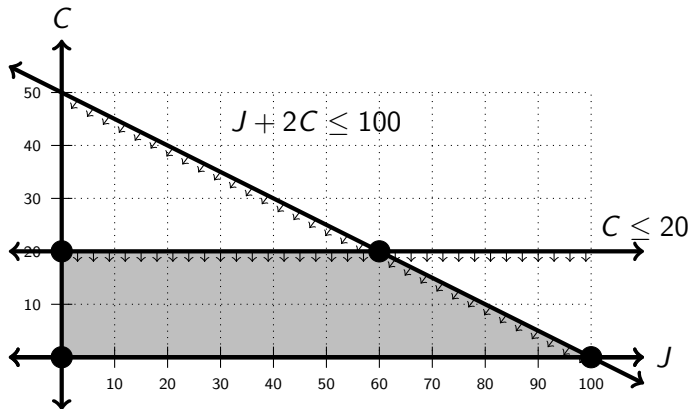
3.1: Graphing systems of inequalities

$200x + 100y \leq 1000$

$x \geq 0$

$(x = 0, y = 100)$

$(x = 5, y = 0)$

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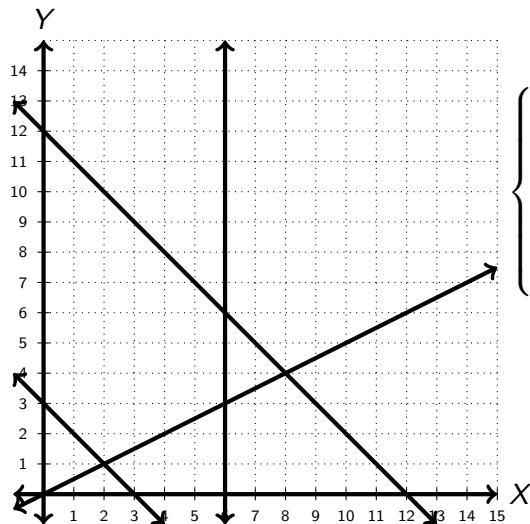
3.1: Graphing systems of inequalities

- Graph each equality (line)

- Figure out which side of the line is good

- Shade the region that is on the correct side of **all** lines

- Alternatively: figure out which of the pieces is good

3.1: Graphing systems of inequalities

Draw little arrows to show which side is good.
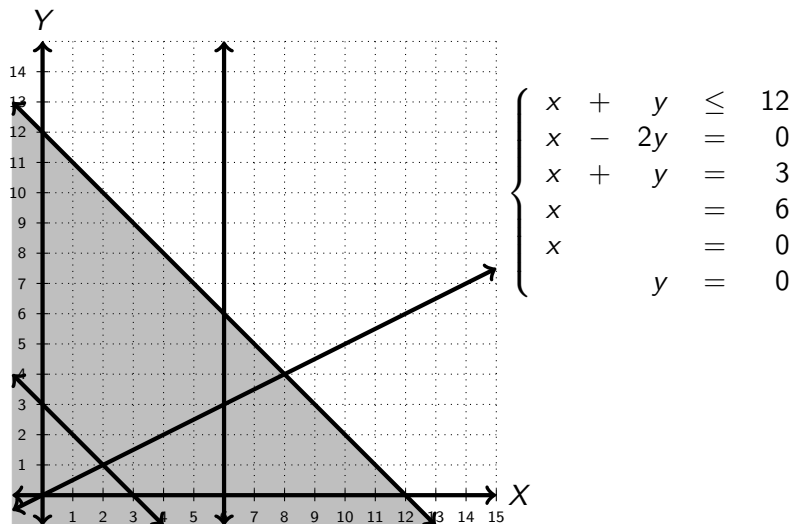
3.1: Graphing systems of inequalities

$$\begin{cases} x & + & y & = & 12 \\ x & - & 2y & = & 0 \\ x & + & y & = & 3 \\ x & & & = & 6 \\ x & & & = & 0 \\ & & y & = & 0 \end{cases}$$

Draw all the lines, then check each inequality.

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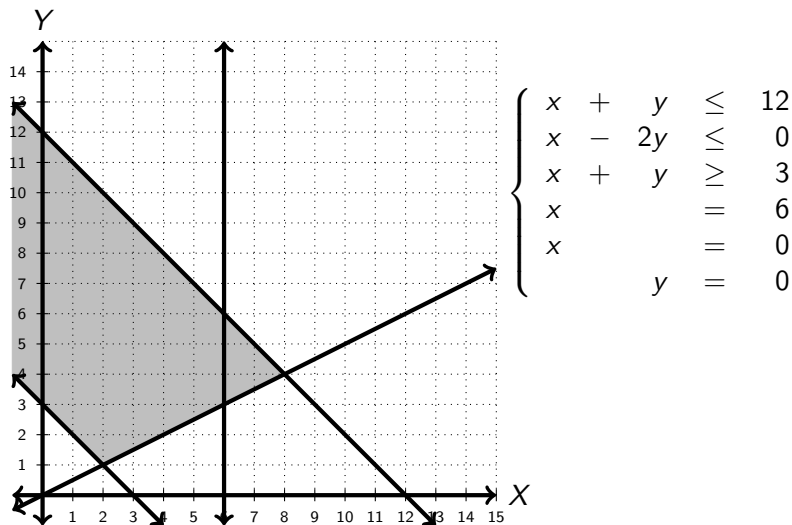
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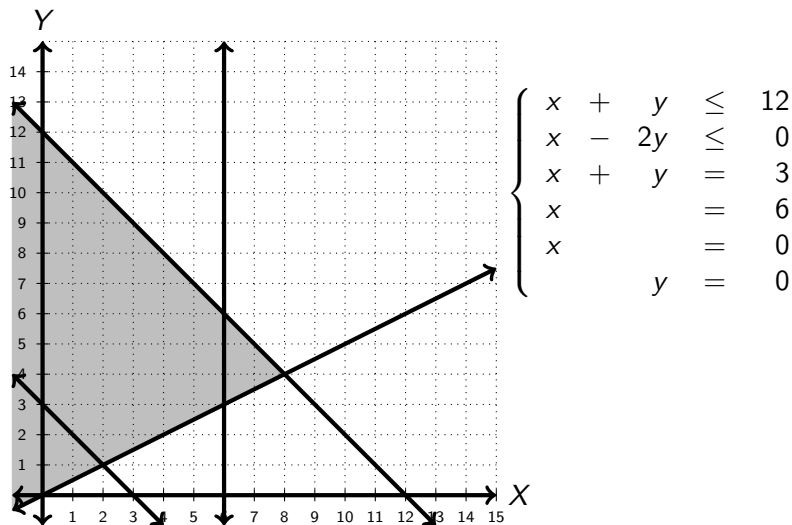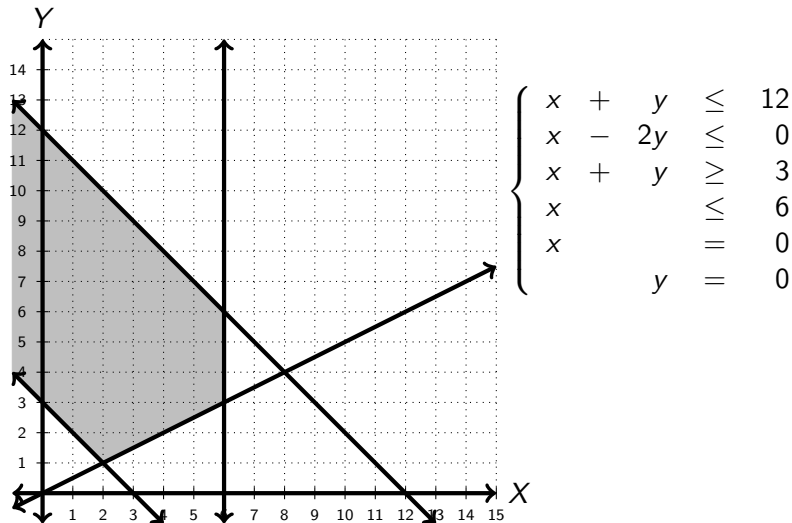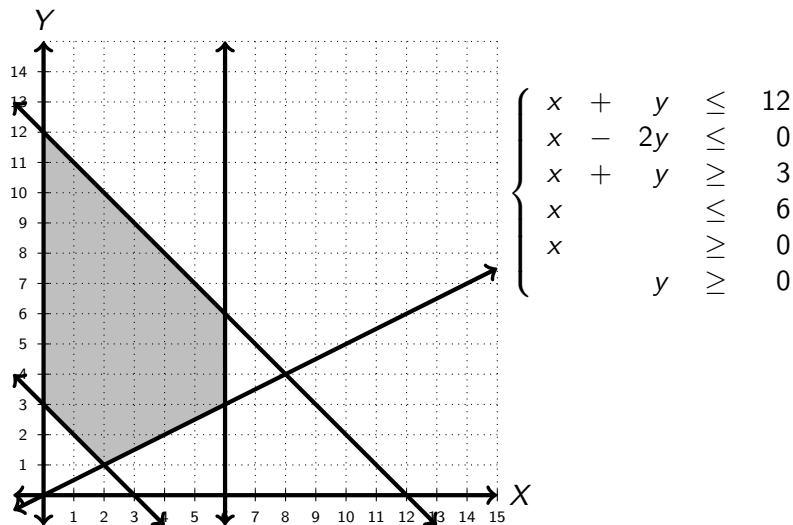
Draw all the lines, then check each inequality. Too many regions!

3.1: Graphing systems of inequalities
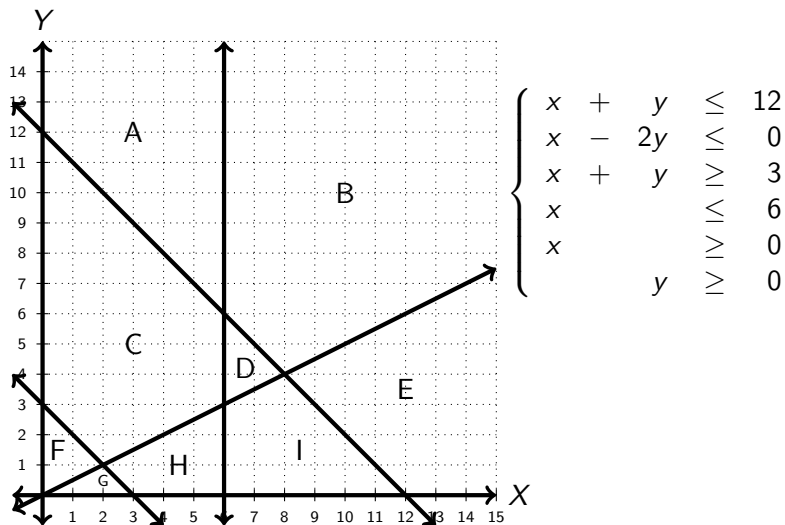
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Check a point in each region to find the right one.

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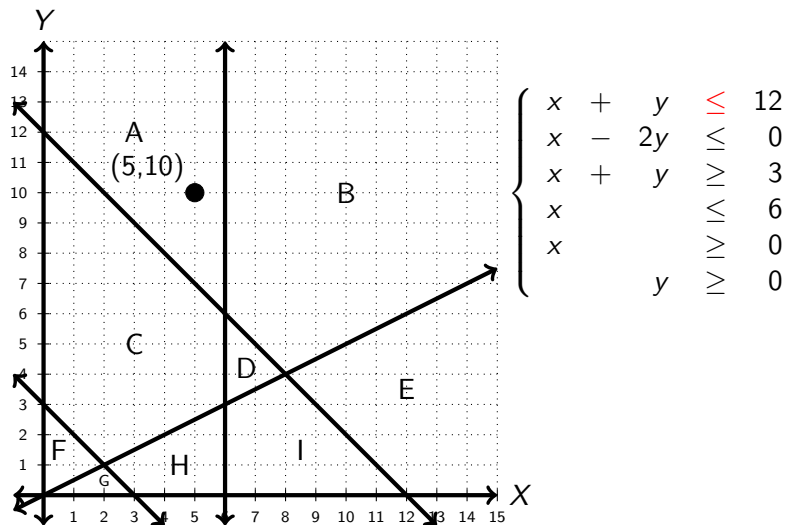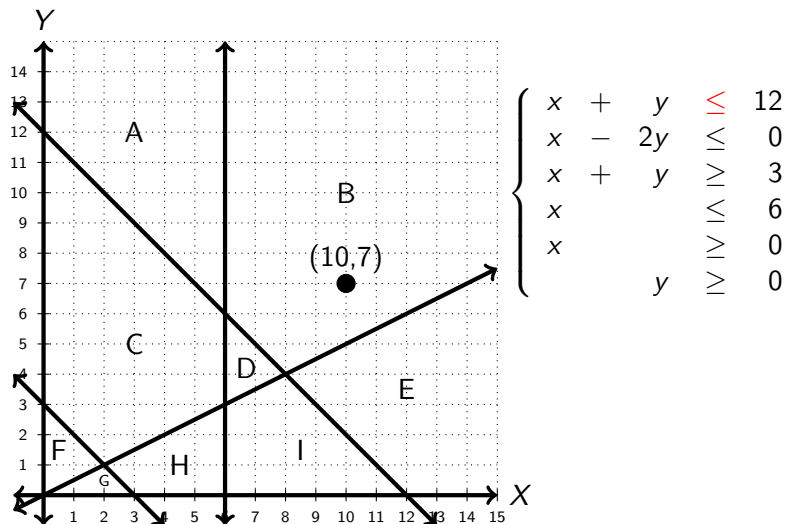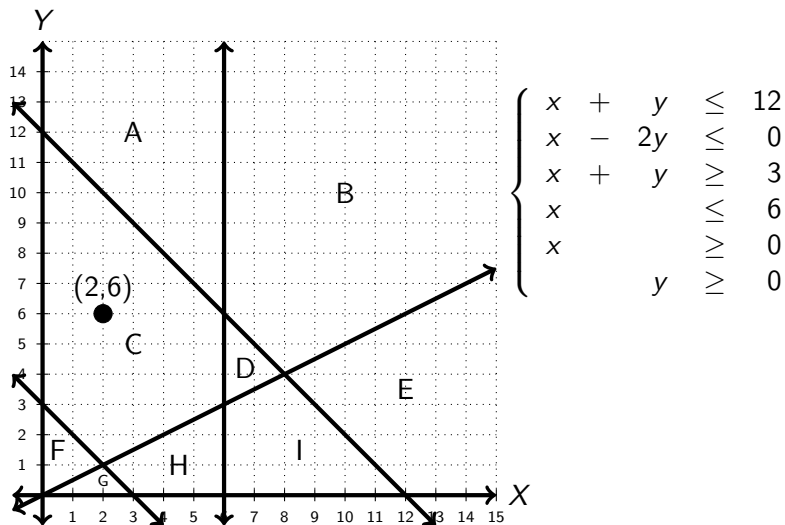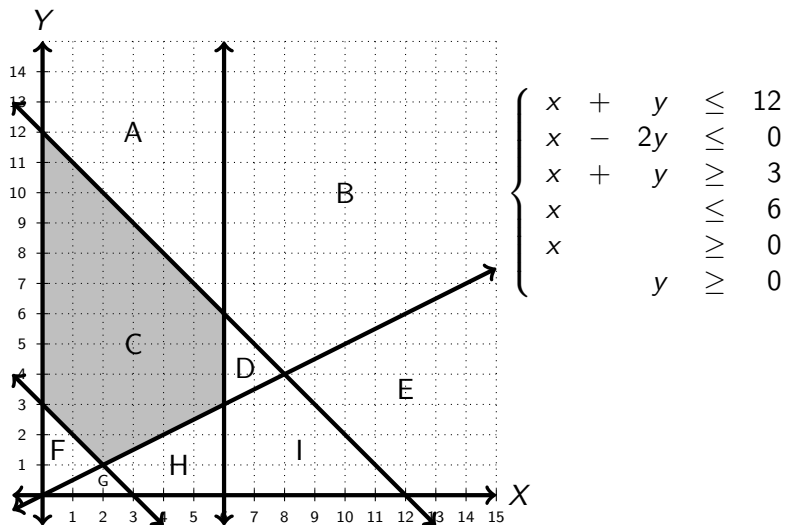
Check a point in each region to find the right one. Yay!

3.1: Finding corners

$$\begin{cases} x & + & y & \leq & 12 \\ x & - & 2y & \leq & 0 \\ x & + & y & \geq & 3 \\ x & & & \leq & 6 \\ x & & & \geq & 0 \\ & & y & \geq & 0 \end{cases}$$
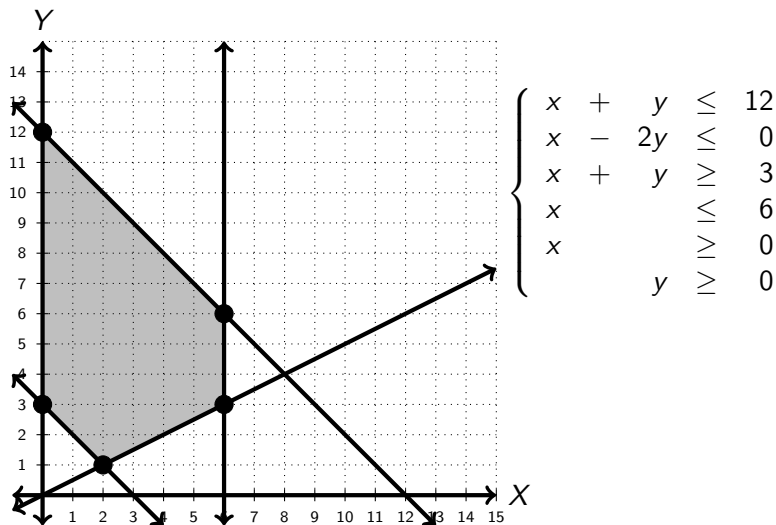
Intersect each pair of lines, and check it satisfies other inequalities

3.1: Finding corners

- For each pair of lines, find the intersection

- Then check that intersection satisfies the rest of the inequalities

- Not all intersections are corners!

- All corners are intersections.

- Intersections are just 2×3 RREF problems!

3.1: How many corners are there?

- How many angles does a triangle have?

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- How many angles does a triangle have?

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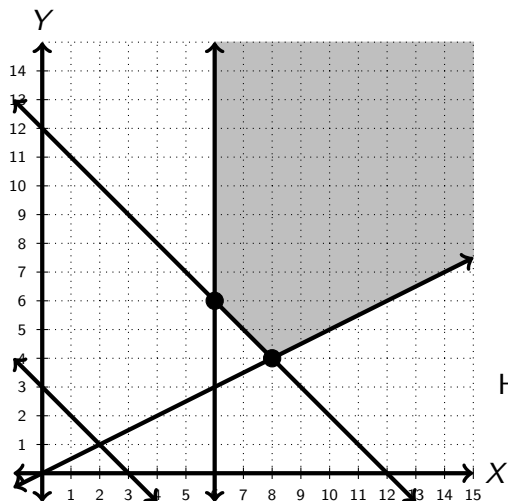
- An *n*-sided polygon has *n* angles too!

3.1: Where's the missing corner?
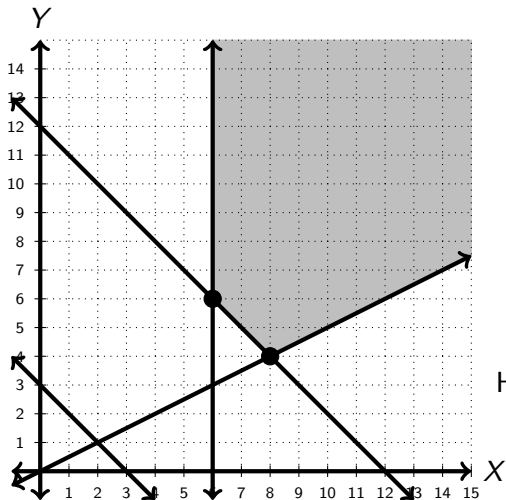
How many edges?

How many corners?

3.1: Where's the missing corner?

How many edges?

How many corners?

- This is called **unbounded** and it means we need to handle the "missing corner" specially.