MA162: Finite mathematics

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SCHEDULE:

- HW 3.2-3.3 extended to Tuesday Oct 11th, 2011.
- HW 4.1-4.2 due Mondy Oct 17th, 2011.
- Exam 2 is Monday, Oct 17th, 2011, in CB106.

Today we will cover 4.1: Simplex algorithm

Exam 2: Overview

- 50% Ch. 3, Linear optimization with 2 variables
 - Graphing linear inequalities
 - ② Setting up linear programming problems
 - 3 Method of corners to find optimum values of linear objectives
- 50% Ch. 4, Linear optimization with millions of variables
 - I Slack variables give us flexibility in RREF
 - 2 Some RREFs are better (business decisions) than others
 - 3 Simplex algorithm to find the best one using row ops
 - ④ Accountants and entrepreneurs are two sides of the same coin

4.1: Linear programming problems

- An LPP has three parts:
 - The variables (the business decision to be made)
 - The inequalities (the laws, constraints, rules, and regulations)
 - The objective (maximize profit, minimize cost)
- If there are more than two variables, graphing gets hard!
- We need to use matrices to handle so many variables.

- Old MacDonald had 100 acres, \$6000, and 2400 labor hours
- Crop A costs him \$50/acre and 20hrs/acre in labor
- Crop B costs him \$60/acre and 25hrs/acre in labor
- Crop A earns him \$150/acre and Crop B earns him \$200/acre
- How many acres of each crop should he plant?

4.1: Today's LPP in summary form

- Variables:
 - $X \hspace{.1in} = \hspace{.1in} \mathsf{Number of acres of crop A to plant}$
 - Y =Number of acres of crop B to plant
- Constraints:

X +	$Y \leq$	100	Land
50X + 6	$60Y \leq 0$	6000	Capital
20X + 2	$25Y \leq 2$	2400	Labor

• Objective:

Maximize P = 150X + 200Y

4.1: Inequalities are just equalities in disguise

- All of our variables are non-negative (realistic)
- Think about $X + Y \leq 100$
- It means X + Y has not yet exceeded 100
- We could still add something to bump it up to 100
- X + Y + U = 100 with $X, Y, U \ge 0$
- U is "the rest" of the 100, the "unused" land, the **slack**

$$U = 100 - (X + Y)$$

• "U = 20" means we left 20 acres fallow (unused)

4.1: LPP are just systems of equations in disguise

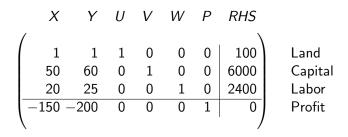
- $\begin{array}{rrrr} X + & Y \leq & 100 & \text{Land} \\ 50X + & 60Y \leq & 6000 & \text{Capital} \\ 20X + & 25Y \leq & 2400 & \text{Labor} \end{array}$
- Define our slack variables:
 - U = 100 (X + Y) unused Land V = 6000 - (50X + 60Y) unused Capital W = 2400 - (20X + 25Y) unused Labor
- Even profit is an equation: P = 150X + 200Y just means -150X - 200Y + P = 0
- Now we have a system of equations:

$$\begin{array}{cccc} X + & Y + & U = & 100 & \text{Land} \\ 50X + & 60Y + & V = & 6000 & \text{Capital} \\ \hline 20X + & 25Y + & W = & 2400 & \text{Labor} \\ \hline -150X - & 200Y + & P = & 0 & \text{Profit} \end{array}$$

- 4.1: Write it as a matrix
 - The system of equations:

X +	Y + U =	100	Land
50X +	60Y + V =	6000	Capital
20X +	25Y + W =	2400	Labor
-150X -	200Y + P =	0	Profit

Now as a matrix:



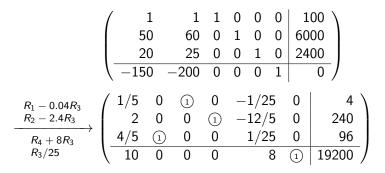
4.1: Analyze the matrix

• Our matrix is basically in RREF!

- U,V,W,P have pivots, and X,Y are free.
 We can make X and Y whatever we want!
- Wait. What do we want them to be? Isn't that why we started doing this anyways?
- OMG We have the wrong pivots!

4.1: Operation Row Op to the rescue!

- Well, what if we decided the 25 should have been a pivot?
- Easy to fix using row ops:



• Y, U, V, P are pivots, X and W are free.

4.1: What does it say now?

• First row says U = 4 - (1/5)X + (1/25)W

- Last row says P = 19200 10X 8W
- X and W are free, what should they be?
- Every acre of crop A we plant costs us \$10!

4.1: Summary

	Χ	Y	U	V	W	Р	RHS	
4	/5 2 /5	0 0 ①	① 0 0	0 ① 0	$-1/25 \\ -12/5 \\ 1/25 \\ 8$	0 0 0	4 240 96 19200	Land Capital Crop B Profit
							/	

- Our free variables are X = 0 and W = 0Plant no acres of crop A, and use all available labor
- First row says U = 4 (1/5)X + (1/25)W = 4Leave 4 acres fallow
- Second row says V = 240 2X + (12/5)W = 240
 Leave \$240 unspent
- Third row says Y = 96 (4/5)X (1/25)W = 96Plant 96 acres of crop B
- Last row says P = 19200 10X 8W = 19200
 Profit is \$19,200

4.1: The key was choosing the right pivot

- How did we know 25 was a good pivot?
- In U = 100 X Y, if we make Y too big, U goes negative
- "too big" is complicated
- If a variable is free, the only safe bet for its value is 0
- But we had P = 150X + 200Y, Y = 0 is cowardly
- We needed to make Y a pivot, rather than Y being free
- So we want a pivot in the Y column.

4.1: Choosing the pivot

- Choose the pivot column first: any column with a negative number at the bottom is OK "Leftmost" and "Most negative" are reasonable strategies
- Which row?
- Each row has a maximum allowed Y: U = 100 - X - Y allows $Y \le 100$ V = 6000 - 50X - 60Y allows $Y \le 100$ W = 2400 - 20X - 25Y allows $Y \le 2400/25 = 96$
- The W row is most restrictive, so we use it
- After you pick the pivot column, choose the pivot row by computing these ratios

Choose the smallest non-negative ratio

4.1: Once is not enough

- Usually a single pivot change is not enough
- The bottom row may still have negatives
- Just choose a pivot again, and repeat
- Make sure the right-hand-sides are always non-negative
- If they are negative, problem is harder or you've made a mistake
- See the silly webpage http://www.ms.uky.edu/~jack/2011-08-MA162/ch4.html