MA162: Finite mathematics

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Schedule:

- HW 3.2-3.3 extended to Tuesday Oct 11th, 2011.
- HW 4.1-4.2 due Mondy Oct 17th, 2011.
- Exam 2 is Monday, Oct 17th, 2011, in CB106.

Today we will review 4.1: Simplex algorithm

Exam 2: Overview

- 50% Ch. 3, Linear optimization with 2 variables
  1. Graphing linear inequalities
  2. Setting up linear programming problems
  3. Method of corners to find optimum values of linear objectives
- 50% Ch. 4, Linear optimization with millions of variables
  1. Slack variables give us flexibility in RREF
  2. Some RREFs are better (business decisions) than others
  3. Simplex algorithm to find the best one using row ops
  4. Accountants and entrepreneurs are two sides of the same coin

4.1: Linear programming problems

- An LPP has three parts:
  - The variables (the business decision to be made)
  - The inequalities (the laws, constraints, rules, and regulations)
  - The objective (maximize profit, minimize cost)
- We replace inequalities with equalities using slack variables
- Then we choose which columns are pivots to get the best RREF
- We set free variables to 0 to get our basic solution

Practice Exam: #1 Practical

- Paul E. Titian is running for SG. To gain popular support, Paul is putting together two gift bags most simply described as the Cheap and the Tasty. Paul's generous patrons have donated 200 packets of diet tea mix, 96 cans of energy drink, 200 fun size candy bars, and 800 mints. Paul expects votes from about 10% of the people he gives the Cheap, and from about 40% of the people he gives the Tasty. How many bags of each type should Paul give away in order to maximize his support?

	Tea	Energy	Candy	Mint	Voters
Cheap	2	1	1	10	10%
Tasty	2	1	5	2	40%
Total	200	96	200	800	

Paul's long time advisor Gesse McGee suggests that Paul produce 76 Cheap bags, and 20 Tasty bags. What would be the result of Gesse's advice? Can you do better?

Practice exam: #2 Setup

- Jumble Juice wants you to set their production goals. They use pineapple concentrate (PC), orange juice concentrate (OJC), and banana pulp (BP) to make their three products: Pineapple-Orange (PO), Orange-Banana (OB), and Pineapple-Orange-Banana (POB). Jumble Juice expects to make a profit of $1.00 per unit of PO, $0.80 per unit of OB, and $0.90 per unit of POB. They can sell up to 1500 units of PO, 500 units of OB, and 1500 units of POB. How much of each blend should be shipped in order to maximize profit under the current inventory constraints and without exceeding the current demand?

	P	O	B	Demand	Profit
PO	8	8	0	1500	$1.00
OB	0	12	4	500	$0.80
POB	4	8	4	1500	$0.90
Available	16000	24000	5000		

Practice exam: #2 Answer

- PO, OB, and POB are the number of units of each product to mix (Let PC, OJC, and BP be the amount of unused ingredients, and Let POD, OBD, POBD be the number of customers still demanding each juice mix)

- Then our constraints are:

$$\begin{aligned} PC: &\quad 8(PO) + 0 + 4(POB) \leq 16000 \\ OJC: &\quad 8(PO) + 12(OB) + 8(POB) \leq 24000 \\ BP: &\quad 0 + 4(OB) + 4(POB) \leq 5000 \end{aligned}$$

  and $0 \leq PO \leq 1500$, $0 \leq OB \leq 500$, $0 \leq POB \leq 1500$

- Our objective is to maximize the profit,

  $1.00(PO) + 0.80(OB) + 0.90(POB)$

Practice exam: #3 Inequalities to tableau

- Practice exam just has some made-up inequalities:

Maximize $P = x + 2y + 3z$ subject to:

$$\begin{aligned} 4x + 5y + 3z &\leq 1001, \\ 6x + 7y + 8z &\leq 1002, \\ 11x + 10y + 9z &\leq 1003, \\ 13x + 14y + 12z &\leq 1004, \\ 15x + 17y + 16z &\leq 1005, \\ x, y, z &\geq 0 \end{aligned}$$

X	Y	Z	U	V	W	S	T	P	RHS
4	5	3	1	0	0	0	0	0	1001
6	7	8	0	1	0	0	0	0	1002
11	10	9	0	0	1	0	0	0	1003
13	14	12	0	0	0	1	0	0	1004
15	17	16	0	0	0	0	1	0	1005
-1	-2	-3	0	0	0	0	0	1	0

Practice Exam: #3 for Jumble juice

- Convert the Jumble Juice setup to a Tableau
- PC, OJC, BP are straightforward
- POD is the unsatisfied demand. The customers want 1500 PO, so

$$POD = 1500 - PO \qquad PO + POD = 1500$$

- The tableau is:

PO	OB	POB	PC	OJC	BP	POD	OBD	POBD	Profit	RHS
8	0	4	1	0	0	0	0	0	0	16000
8	12	8	0	1	0	0	0	0	0	24000
4	8	4	0	0	1	0	0	0	0	5000
1	0	0	0	0	0	1	0	0	0	1500
0	1	0	0	0	0	0	1	0	0	500
0	0	1	0	0	0	0	0	1	0	1500
-1.00	-0.80	-0.90	0	0	0	0	0	0	1	0

Practice Exam: #4 One step of simplex algorithm

- One step means:
  1. Find the pivot column (look for negative in bottom row)
  2. Find the pivot row (calculate the ratios)
  3. Make the pivot 1 with $R_i/$ whatever
  4. Zero the numbers above and below with $R_j - (t/a)R_i$ row ops

Practice Exam: #5 Read the answer

- Read the answer and give a plain English recommendation

PO	*OB*	*POB*	*PC*	*OJC*	*BP*	*POD*	*OBD*	*POBD*	*P*	*RHS*
1	0	0	0	0	0	1	0	0	0	1500
0	0	0	1	1	-3	-16	0	0	0	1000
0	0	1	$1/4$	0	0	-2	0	0	0	1000
0	1	0	$-1/4$	0	$1/4$	2	0	0	0	250
0	0	0	$1/4$	0	$-1/4$	-2	1	0	0	250
0	0	0	$-1/4$	0	0	2	0	1	0	500
0	0	0	$1/40$	0	$1/5$	$4/5$	0	0	1	2600

- This is just an RREF, read it like on exam 1

Practice Exam: #5 Read the answer

- PO = 1500, OB = 500, POB = 1000
- PC = 0, OJC = 1000, BP = 0
- POD = 0, OBD = 250, POBD = 500
- Profit = \$2600
- Plain English:
  Make 1500 units of PO, 500 units of OB, and 1000 units of POB. This maximizes profit at \$2600. Be warned that you'll have leftover Orange Juice Concentrate, and 750 units of unmet demand, so what you really need is Banana Pulp and Pineapple concentrate.