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MA111
Ch. 1 Exam Review
2011-09-09

Instructions: No books or notes may be used on this exam. You will have 50 minutes to answer all of the following questions. Additional paper is available if necessary. Please write legibly and keep your paper as organized as possible. Show all of your work! Answers without work or explanation will not receive full credit. Please use complete sentences where appropriate to explain your responses. Each part is worth 25% of the exam.

Part I: Vocabulary

Match the type of winner to the description, by writing the letter of the definition next to the term.

A Majority winner — More than half 1st Place

B Condorcet winner — All Head-to-Head

C Plurality winner — Most 1st Place

D Borda Count winner — Best Average

E Plurality w/elimination winner

F Pairwise comparison winner — Most Head-to-Head

(A) A candidate with more than 50% of the first place votes

(B) A candidate who outranks every other candidate head-to-head

(C) A candidate with the most first place votes

(D) Each first place vote gets the most points, one less point for second place votes, etc. Most total points wins.

(E) The last surviving candidate during the game: the candidate with the least first place votes is eliminated, repeat.

(F) A candidate that wins more head-to-head matchups than any other candidate

Part II: Vote counting

For each method, show how one calculates the winner based on the following preference schedule. Make sure to clearly indicate which candidate is the final winner for each method. Merely giving the winner without showing how the winner was decided receives **no** credit.

	7	6	5	3
1st	D	B	A	C
2nd	A	C	C	D
3rd	C	A	B	B
4th	B	D	D	A

- Plurality: D wins

D: 7
B: 6
A: 5
C: 3

- Borda: C wins

A: 7(3) + 6(2) + 5(4) + 3(1) = 56
B: 7(1) + 6(4) + 5(2) + 3(2) = 47
C: 7(2) + 6(3) + 5(3) + 3(4) = 59
D: 7(4) + 6(1) + 5(1) + 3(3) = 48

- Plurality with elimination:

C is eliminated with 3

7	6	5	3
D	B	A	D

A is eliminated with 5

7	6	5	3
D	B	B	D

D is eliminated with 10

7	6	5	3
B	B	B	B

B wins!

- Pairwise comparison:

A vs B	
7	6
5	3

A vs C	
7	6
5	3

A vs D	
6	7
5	3

B vs C	
6	7
	5
	3

B vs D	
6	7
5	3

C vs D	
6	7
5	
3	

A is Condorcet!

	W	L	T
A:	3	0	0
B:	1	2	0
C:	2	1	0
D:	0	3	0

A wins with 3 points

Part III: Strategic Voting

In each of the following elections[1], This election is decided by the stated method, and the voters' secret feelings are revealed to the right. Answer the following three questions for each:

(a) If everyone votes honestly, who wins?

(b) Which candidates would the voters from the first column prefer?

(c) Can the first column of voters lie to change the outcome to one of their preferred candidates?

Plurality:

(a) C wins with 10 1st place votes

(b) A or B are both better

(c) No. C has a majority without those 8 voters.

	8	7	3
1st	A	C	C
2nd	B	B	A
3rd	C	A	B

Borda count

(a) C wins with 38 points

(b) A or B are both better

A: 8(3) + 7(1) + 3(2) = 37
B: 8(2) + 7(2) + 3(1) = 33
C: 8(1) + 7(3) + 3(3) = 38

(c) Yes. They cannot help A any better, but they can help B get 8 more points to win.

A: 8(2) + 7(1) + 3(2) = 37 − 8
B: 8(3) + 7(2) + 3(1) = 33 + 8
C: 8(1) + 7(3) + 3(3) = 38

	8	7	3
1st	A	C	C
2nd	B	B	A
3rd	C	A	B

8	7	3
B	C	C
A	B	A
C	A	B

Plurality with elimination

(a) A is eliminated, $\frac{2\ 9\ 8\ 7}{C\ B\ B\ C}$, then B wins

(b) A or C are both better.

(c) Yes. They can't help C, but they can help A: C is eliminated, $\frac{2\ 9\ 8\ 7}{A\ B\ A\ A}$, then A wins

	2	9	8	7
1st	C	B	A	C
2nd	A	C	B	A
3rd	B	A	C	B

2	9	8	7
A	B	A	C
C	C	B	A
B	A	C	B

Pairwise comparisons

(a) A wins with 2.5 pts

(b) Only B is better.

(c) They can make A look worse, so that B wins with 2 pts (A knocked down to 1 pt)

A vs B: 6, 9, 3
A vs C: 9, 6, 3
A vs D: 9, 6
B vs C: 9, 3
B vs D: 9, 6, 3
C vs D: 9, 6

	9	6	3
1st	B	D	A
2nd	A	A	C
3rd	C	B	D
4th	D	C	B

	W	L	T
A	2	0	1
B	1	0	2
C	1	2	0
D	0	2	1

A vs B: 6, 9, 3
A vs C: 6, 9, 3
A vs D: 9, 6

9	6	3
B	D	A
C	A	C
D	B	D
A	C	B

	W	L	T
A	0	1	2
B	1	0	2
C	1	1	1
D	1	1	1

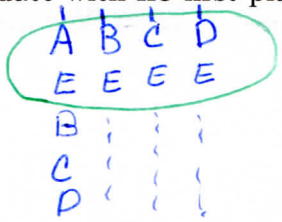
[1] There will only be 2 elections on the real exam, but the questions might be a little different.

Part IV: Fairness

Explain your reasoning very carefully. If it **can** happen, then give an example where it **does** happen. It it cannot, then explain clearly why it is impossible.[2]

- Can a candidate with **no** first place votes be a Condorcet winner?

Yes!

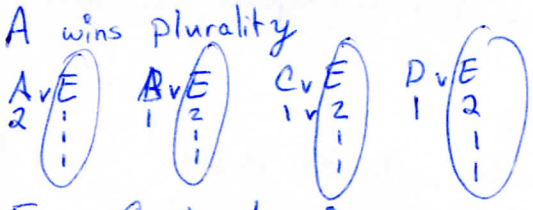


In any head to head with E, E gets 3 votes and the other guy gets 1.

- Can a Condorcet winner lose an election decided by plurality?

Yes!

A wins plurality



E is Condorcet winner

- Can a majority winner lose an election decided by plurality?

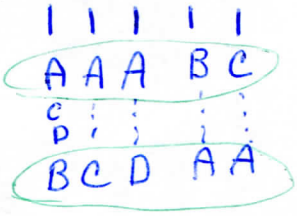
No! If you have more than half, everyone else has to share less than half, so they have less than you.

- Can a majority winner have more than 50% of the last place votes?

No! If more than half like you best, the only people left to hate you are less than half.

- Can a majority winner have the most last place votes?

Yes!



A only has 2/5 last place votes, but B, C, D have only 1/5 each.

[2] Two of these five questions will be on the exam. The last two questions are due to Amadeusz Pyrek.