

Name: _____

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MA111
Ch. 1 Exam
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Instructions: No books or notes may be used on this exam. You will have 50 minutes to answer all of the following questions. Additional paper is available if necessary. Please write legibly and keep your paper as organized as possible. Show all of your work! Answers without work or explanation will not receive full credit. Please use complete sentences where appropriate to explain your responses. Each part is worth 25% of the exam.

Part I: Vocabulary

Match the type of winner to the description, by writing the letter of the definition next to the term.

- | | | |
|----------|--|----------------------------------|
| <u>C</u> | Majority winner | More than half first place votes |
| <u>B</u> | Condorcet winner | All Head to Head Wins |
| <u>E</u> | Plurality winner | Most first place votes |
| <u>A</u> | Borda Count winner | Best Average |
| <u>F</u> | Plurality w/ <u>elimination</u> winner | |
| <u>D</u> | Pairwise comparison winner | Most Head to Head wins |

- (A) Each first place vote gets the most points, one less point for second place votes, etc. Most total points wins.
- (B) A candidate who outranks every other candidate head-to-head
- (C) A candidate with more than 50% of the first place votes
- (D) A candidate that wins more head-to-head matchups than any other candidate. Specifically, a head-to-head victory is worth one point, and a head-to-head tie is worth 0.5 points. Most points wins.
- (E) A candidate with the most first place votes
- (F) The last surviving candidate during the game: the candidate with the least first place votes is eliminated, repeat.

Part II: Vote counting

For each method, show how one calculates the winner based on the following preference schedule. Make sure to clearly indicate which candidate is the final winner for each method. Merely giving the winner without showing how the winner was decided receives **no** credit.

	7	6	5	3
1st	C	A	D	B
2nd	D	B	B	C
3rd	B	D	A	A
4th	A	C	C	D

- Plurality:

C wins with 7 first place votes

A: 6

B: 3

C: 7

D: 5

- Borda:

B wins with 59 Borda points

$$A: 7(1) + 6(4) + 5(2) + 3(2) = 47$$

$$B: 7(2) + 6(3) + 5(3) + 3(4) = 59$$

$$C: 7(4) + 6(1) + 5(1) + 3(3) = 48$$

$$D: 7(3) + 6(2) + 5(4) + 3(1) = 56$$

- Plurality with elimination:

B is eliminated with 3 votes

D is eliminated with 5 votes

C is eliminated with 10 votes

A wins!

7	6	5	3
C	A	D	C
7	6	5	3
C	A	B	A
7	6	5	3
A	A	A	A

- Pairwise comparison:

D wins with 3 points

D is a Condorcet winner too!

A vs B	A vs C	A vs D	B vs C	B vs D	C vs D
6 7 5 3	6 7 5 3	6 7 5 3	6 7 5 3	6 7 5 3	7 6 3 5

	Win	Loss	Tie	Total
A	1	2	0	1
B	2	1	0	2
C	0	3	0	0
D	3	0	0	3

Part III: Strategic Voting

In both of the following elections, the election is decided by the stated method, and the voters' secret feelings are revealed to the right. Answer the following three questions for both:

- If everyone votes honestly, who wins?
- Which candidates would the voters from the first column prefer?
- Can the first column of voters lie to change the outcome to one of their preferred candidates? If they can, then show how. If not, then say why.

Plurality:

- Z wins with 10 first place votes

- X or Y are both better

- No! Z has a majority regardless of those 8 voters

	8	7	3
1st	X	Z	Z
2nd	Y	Y	X
3rd	Z	X	Y

Plurality with elimination

- X is eliminated 2 9 8 7 Z Y Y Z, then Y wins

- Z is better than Y or X is

- Cannot directly help Z, but can help X.

Z is eliminated is, but then

2 9 8 7
X Y X X

X wins!

	2	9	8	7
1st	Z	Y	X	Z
2nd	X	Z	Y	X
3rd	Y	X	Z	Y

	2	9	8	7
1st	X	Y	X	Z
2nd	Z	Z	Y	X
3rd	Y	X	Z	Y

Part IV: Fairness

Explain your reasoning very carefully. If it **can** happen, then give an example where it **does** happen. If it cannot, then explain clearly why it is impossible.

- Can a Condorcet winner lose an election decided by plurality?

Yes!

	2	1	1	1
A	B	C	D	
B	A	B	B	
C	C	A	A	
D	D	D	C	

A wins Plurality, but

B vs A 1 1 1	A 2	B vs C 2 1 1	B vs D 2 1 1
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B is Condorcet winner.

- Can a majority winner have the most last place votes?

Yes!

	1	1	1	2
A	A	A	A	D
B	C	D	C	
C	B	B	B	
D	B	C	A	

A has $3/5$ (more than half) of first place votes
A has $2/5$ (more than $1/5$, $1/5$, or $1/5$) of last place votes