#### MA111: Contemporary mathematics

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Schedule:

- HW 1A,1B are due Friday, Jan 20th, 2012.
- HW 1C,1D,1E,1G are due Friday, Jan 27th, 2012.
- Exam 1 is Monday, Jan 30th, during class.

Today we will look at how vote counters can get more out of rankings.

- Counting just first place votes can elect a majority loser
- The majority can do something about it; they can lie about who they think is best
- Anytime there is a non-unanimous vote, somebody loses
- Can we encourage people to tell the truth when they vote?
- Can we take into account more than just first place votes?

•	Explain	why A	is the	undisputed	champeen:
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13	12	6	3
В	D	А	D
С	А	С	С
Α	С	В	В
D	В	D	А

• Explain why A is the undisputed champeen:

• Is there any candidate that can beat A?



		13	12	6	3
		В	D	А	D
• E>	plain why A is the undisputed champeen:	С	А	С	С
		А	С	В	В
		D	R	D	Δ

- Is there **any** candidate that can beat A?
- Between A and D, the first and third columns prefer A, that is 19 to 15.
  Between A and C, the second and third columns prefer A, that is 18 to 16.
  Between A and B, the second and third columns prefer A, that is 18 to 16.
- A is called a **Condorcet winner** because it beats every other candidate in a head-to-head matchup. It is better than all the rest. Surely it should win?

	13	31	.2	6	3
	E	3	D	А	D
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	A	١	С	В	В
	Г	)	R	D	Δ

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- A is called a **Condorcet winner** because it beats every other candidate in a head-to-head matchup. It is better than all the rest. Surely it should win?
- **Condorcet fairness criteria** simply says a Condorcet winner should, you know, actually win the election

Activity: Let's keep score



- D just killed in the second and fourth columns, beating (12)(1+1+1)+(3)(1+1+1) candidates. Of course that is it.
- How many candidates did C beat? How about B and A?

Activity: Let's keep score



- D just killed in the second and fourth columns, beating (12)(1+1+1)+(3)(1+1+1) candidates. Of course that is it.
- How many candidates did C beat? How about B and A?
- Well C has the high score, so C is the Borda winner

- In track-and-field, swimming, and similar sports, we have lots of rankings
- Maybe Alex has one gold and three bronze and Bob has four silvers
- Who did "better"? Can we choose a winner?
- Do we just count gold medals, or should we count the silvers and the bronze? In Borda count, we count them all, down to last place.
- In Borda count everyone gets a trophy, so its more fair.

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- Do they always let the Condorcet winner win?

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• What happens in this election?

	10	6
1st	A	В
2nd	В	С
3rd	C	А

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• What happens in this election?

- A has more than 50% of the first place vote, A is a majority winner.
- A beats B 10 to 6, and A beats C 10 to 6, so A is also a Condorcet winner.

	10	6
1st	А	В
2nd	В	С
3rd	С	А

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- Do they always let the Condorcet winner win?

• What happens in this election?

- A has more than 50% of the first place vote, A is a majority winner.
- A beats B 10 to 6, and A beats C 10 to 6, so A is also a Condorcet winner.
- But A only beat (10)(1+1) + (6)(0) = 20 candidates, while B beat (10)(1) + (6)(1+1) = 22 candidates. B has the high score, B is the **Borda winner**

	10	6
1st	Α	В
2nd	В	С
3rd	C	А

#### Winners and fairness

- Majority winner has more than 50% of the first place votes
- **Plurality winner** has the most first place votes
- Condorcet winner beats all other candidates one-on-one
- **Borda winner** beats more candidates than any other candidate (high score)

- Majority fairness criterion says a Majority winner should win
- Condorcet fairness criterion says a Condorcet winner should win

- Vocab: Majority winner, Condorcet winner, Plurality winner, Borda winner
- Read section 1.3 of the textbook. (Should have already read 1.1-1.2.)
- 1A and 1B is due today.
- I'll be in the Mathskeller from 2pm until dinner time. (CB63; basement of this building; behind the big red doors)

# Further Reading

- Since multiple candidates can dilute the race, candidates might form coalitions ("parties")
- What does "Condorcet" look like with parties?
- A "Smith set" is a party where every candidate in the party can beat every candidate not in the party And is the unique smallest party where that happens
- The "Smith criterion" merely asks that the winner be a member of the Smith set
- We have two voting methods. Do they have strategies?
- The Gibbard-Satterwaite theorem and the Duggan-Schwartz theorem say they do!