# MA111: Contemporary mathematics

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Schedule:

- HW 1A,1B was due Friday, Jan 20th, 2012.
- HW 1C,1D,1E,1G are due Friday, Jan 27th, 2012.
- Exam 1 is Monday, Jan 30th, during class.
- Should have read 1.1-1.3 already. Read 1.4 today.

Today we will look at how to simplify elections and check whether it is fair.

# Review: Everybody is a winner

- Four kinds of winners: Majority, Condorcet, Plurality, Borda
- Not always a majority or Condorcet winner, but if there is, it seems unfair (or at least strange) for them to lose
- Borda gives everyone points based on how they did; high score wins
- Plurality just counts first place votes

# What happens if we eliminate irrelevant candidates?

- If we just asked people for their first place votes (and they were honest), then C wouldn't get any votes. Why not get rid of C?

| 13 | 12 | 6 | 3 |
|---|---|---|---|
| B | D | A | D |
| C | A | C | C |
| A | C | B | B |
| D | B | D | A |

$\longrightarrow$

| 13 | 12 | 6 | 3 |
|---|---|---|---|
| B | D | A | D |
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- If the polls had only asked first place candidates, then C might not have even realized they had a chance!

# Why not keep eliminating?

- A only got 6 votes, so is not a real contender, right? DELETED.

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- With A gone, the 6 voters swing the election to B, and B wins
- In presidential primaries, candidates often drop out of the race as soon as they (or their financial backers) think they are going to lose
- If the voters themselves were steady in the opinions, then this would result in **plurality with elimination**

# Unfortunately voters change their minds too

- Secret poll before the elimination election. Who will win?

| 10 | 8 | 7 | 4 |
|---|---|---|---|
| A | B | C | C |
| C | A | B | A |
| B | C | A | B |

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| C | A | B | A |
| B | C | A | B |

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|---|---|---|---|
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$\xrightarrow{\text{combine}}$

| 18 | 11 |
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- 4 voters on the end overhear the results, and change their mind:

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|---|---|---|---|
| A | B | C | C |
| C | A | B | A |
| B | C | A | B |

$\xrightarrow{\text{lie}}$

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|---|---|---|---|
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- Now who wins?

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- Now who wins? B wins!
- Voters tried to help A, but made A lose. This violates the **monotonicity criterion**.

# Voter apathy: does my vote even count?

- Plurality with elimination. Who will win based on this poll?

| 7 | 6 | 5 |
|---|---|---|
| A | B | C |
| B | C | A |
| C | A | B |

# Voter apathy: does my vote even count?

- Plurality with elimination. Who will win based on this poll? A

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|---|---|---|
| A | B | C |
| B | C | A |
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$\xrightarrow{\text{C lost}}$

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| A | B | A |
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$\rightarrow$ A wins 12 to 6

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| C | A | B |

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| 7 | 6 | 5 |
|---|---|---|
| A | B | A |
| B | A | B |

$\rightarrow$ A wins 12 to 6

- 2 of B's supporters just give up and don't vote. Who wins?

| 7 | 4 | 5 |
|---|---|---|
| A | B | C |
| B | C | A |
| C | A | B |

# Voter apathy: does my vote even count?

- Plurality with elimination. Who will win based on this poll? A

| 7 | 6 | 5 |
|---|---|---|
| A | B | C |
| B | C | A |
| C | A | B |

$\xrightarrow{\text{C lost}}$

| 7 | 6 | 5 |
|---|---|---|
| A | B | A |
| B | A | B |

$\rightarrow$ A wins 12 to 6

- 2 of B's supporters just give up and don't vote. Who wins? C

| 7 | 4 | 5 |
|---|---|---|
| A | B | C |
| B | C | A |
| C | A | B |

$\xrightarrow{\text{B lost}}$

| 7 | 4 | 5 |
|---|---|---|
| A | C | C |
| B | A | A |

$\rightarrow$ C wins 9 to 7

# Voter apathy: does my vote even count?

- Plurality with elimination. Who will win based on this poll? A

| 7 | 6 | 5 |
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|---|---|---|
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| C | A | B |

$\xrightarrow{\text{B lost}}$

| 7 | 4 | 5 |
|---|---|---|
| A | C | C |
| B | A | A |

$\rightarrow$ C wins 9 to 7

- By not voting at all, they got a better result (their 2nd place pick)

# A goofy Borda count

- For some reason A through Z all got nominated as candidates.
- 9 voters love A, and like B, and C-Z are like so whatever.
- 1 voter is obsessed with B, and decides to lie on his vote and give A his last place vote. What happens?

| 9 | 1 |
|---|---|
| A | B |
| B | C |
| C | D |
| ⋮ | ⋮ |
| Y | Z |
| Z | A |

# The results of one crazy voter

| 9 | 1 |
|---|---|
| A | B |
| B | C |
| C | D |
| ⋮ | ⋮ |
| Y | Z |
| Z | A |

- A gets $(9)(25) = 225$ points, B gets $(9)(24) + 25 = 241$ points
- One (crazy) voter managed to change the entire election!
- This is only possible when the number of candidates is large compared to the number of voters (a pretty silly situation, but one faced by some small clubs)

# One crazy election

- We've seen a lot of different "winners" here:

| 13 | 12 | 6 | 3 |
|---|---|---|---|
| B | D | A | D |
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- Majority: none
- Condorcet: A
- Plurality with elimination: B
- Borda: C
- Plurality: D

# Further reading: Judgement aggregation

- How do rational voters choose their preferred candidate?
- One simple model is that there are "issues"
- Candidates have "platforms" to describe their stance on issues
- Voters have feelings on each issue and vote for the candidate that agrees with them the most
- One voter choosing a candidate is thus summarizing a group preference

# Further reading: Example

- Suppose there are 7 issues that you feel (equally) strongly about
  (Maybe "Foreign policy", "Government spending", "Unemployment", "Civil Liberties", "Education", "Energy", and "Health Care")
- Candidate Al agrees with you on 3 of the 7 issues
  (but Bill and Clint disagree with you on those 3)
- Candidate Bill agrees with you on 2 other issues
  (but Al and Clint disagree with you on those 2)
- Candidate Clint agrees with you on the remaining 2 issues
  (but Al and Bill disagree with you)
- It seems like Al is the best, but actually he **disagrees with you on a majority** of the most important issues!

# Further reading: What can this explain?

- The previous example may explain why voters are dissatisifed with candidates:
- Running the government is a complicated, multi-faceted task
- Hard for two people to agree on all facets
- Hence the best (plurality) might be lousy (majority loser)
- This explains why people will vote for lousy candidates
- Does it explain why lousy candidates win?
- Surely not **every voter** disagrees with **every candidate**?

# Further reading: Ostrogorski's paradox

- Imagine 3 yes/no issues and 5 voters:

| | V1 | V2 | V3 | V4 | V5 | Majority |
|---|---|---|---|---|---|---|
| Budget: Balance/Services | B | S | S | B | B | B |
| Liberty: Security/Freedom | S | F | S | F | F | F |
| Energy: Cheap/Sustainable | S | S | C | C | C | C |

- Majority wants a balanced budget, personal freedom, and cheap energy
- Now imagine two candidates:

| | Al | Bill |
|---|---|---|
| Budget: Balance/Services | B | S |
| Liberty: Security/Freedom | F | S |
| Energy: Cheap/Sustainable | C | S |

- Majority agrees with the majority of Al's platform
- Majority disagrees with the majority of Bill's platform
- Who wins?

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- Majority agrees with the majority of Al's platform
- Majority disagrees with the majority of Bill's platform
- Who wins? V1:Bill, V2:Bill, V3:Bill, V4:Al, V5:Al