

MA111: Contemporary mathematics

Jack Schmidt

University of Kentucky

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SCHEDULE:

- HW 1A,1B was due Friday, Jan 20th, 2012.
- HW 1C,1D,1E,1G are due Friday, Jan 27th, 2012.
- Exam 1 is Monday, Jan 30th, during class.
- Should have read 1.1-1.4 already. Read 1.5 today.

Today we will look at how to fake a Condorcet winner.

Review: Five types of winners at least

- Majority, Condorcet, Plurality, Borda, Plurality with Elimination
- Plurality with Elimination is strange:
voting for someone can hurt their chances to win!

Burlington 2009 Mayoral elections – failure of **monotonicity**

- Not always a Majority winner, but if there is one, then Plurality and Plurality with Elimination pick them
- Not always a Condorcet winner. What if we *need* one?
Can we fake one?

An example of a nearly Condorcet winner

- Is there a Condorcet winner?

	6	5	1
1st	A	D	B
2nd	C	C	C
3rd	D	A	D
4th	B	B	A

- Work in small groups and check.
Quite a few to check, but you could split up the work.

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CvD	6	5	1
1st		D	
2nd	C		C
3rd			
4th			

- C vs. D is 7 to 5, woo C.

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- Work in small groups and check.
Quite a few to check, but you could split up the work.
- C looks pretty good: let's check the matchups:

CvB	6	5	1
1st			B
2nd	C	C	
3rd			
4th			

- C vs. B is 11 to 1, woo C.

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- Is there a Condorcet winner?

	6	5	1
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- Work in small groups and check.
Quite a few to check, but you could split up the work.
- C looks pretty good: let's check the matchups:

CvA	6	5	1
1st	A		
2nd		C	C
3rd			
4th			

- C vs. A is 6 to 6. Yay?

An example of a nearly Condorcet winner

- Is there a Condorcet winner?

	6	5	1
1st	A	D	B
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3rd	D	A	D
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- Work in small groups and check.
Quite a few to check, but you could split up the work.
- C looks pretty good: let's check the matchups:

CvA	6	5	1
1st	A		
2nd		C	C
3rd			
4th			

- C vs. A is 6 to 6. Yay?
- Not quite Condorcet, but still pretty good.

How can we choose the “most Condorcet” winner

- Pretend we think Condorcet winners are the best kind of winners.
- We'd like to say C was the winner since it beat B and D head-to-head, and tied with A. Nobody else did as well.
- We need a set of rules to decide. Not just “he was close”.
- In groups of 4 to 5, determine a set of rules to decide an election that really uses this idea of winning head-to-head matchups
- Be ready to explain the rules and why those rules will **never** make a Condorcet winner suffer the ignominy of defeat.

Ok, let's try the rules out

- Does A win this one:

	99	1
1st	A	B
2nd	B	C
3rd	C	A

Ok, let's try the rules out

- Does A win this one:

	99	1
1st	A	B
2nd	B	C
3rd	C	A

- Does A win this one:

	40	35	25
1st	A	B	C
2nd	B	A	A
3rd	C	C	B

Ok, let's try the rules out

- Does A win this one:

	99	1
1st	A	B
2nd	B	C
3rd	C	A

- Does A win this one:

	40	35	25
1st	A	B	C
2nd	B	A	A
3rd	C	C	B

- Who wins this one:

	40	35	25
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

Condorcet winners and elimination

- A Condorcet winner is an unstoppable force *mano-a-mano*
- What happens if another candidate is eliminated?

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- A Condorcet winner is an unstoppable force *mano-a-mano*
- What happens if another candidate is eliminated?
- Less victims to witness his victory, but still undefeated
- What about someone who wins using today's rules?
Can eliminating a losing candidate create a different winner?

	6	5	1
1st	A	D	B
2nd	C	C	C
3rd	D	A	D
4th	B	B	A

	3	2	1
1st	K	L	M
2nd	J	J	L
3rd	M	M	K
4th	L	K	J

Key concepts for today

- **Pairwise comparison:** Look at all head-to-head matchups.
A candidate scores 1 point per victory, and 0.5 points per tie.

No consolation prize

Most points wins.

- **Independence of irrelevant alternatives:** (It would be nice if) eliminating a losing candidate does not change the winner.
- Read section 1.5 of the textbook.
- Online homework (30%): 1B, 1C, 1D, 1E, 1G are due Friday.

This is Plurality, Borda, Elimination, Head-to-Head, and Misc