MA111: Contemporary mathematics

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February 6, 2012

Schedule:

- Have read 10.1-10.2; read 10.3 today.
- Homework is due regularly; Pearson dates may be too late
- The second exam is Friday, Feb 17th, during class.
- Mathskeller CB63 for help: MWF 2pm-3pm (Jack), TR 9:30-10:30am (Kaichen)

Today we will look at borrowing money for a year, 10.3, compound interest.

10.2: Review of simple interest

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- \$1000 Treasury Bill earning 10% simple interest annually
  Maturity in 6 years, present value is? \$625.00

$$\begin{aligned} F &= \$1000 \\ r &= 0.010 \text{ per year} \\ t &= 6 \text{ years} \\ F &= P + I = P + Prt = P(1 + rt) \\ P &= F/(1 + rt) = \$1000/(1 + 6(0.01)) = \$1000/1.06 = \$625.00 \end{aligned}$$

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I &= F - P = \$100 \\
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- A savings bond doubles its value in 20 years.
  What is the simple interest APR? 5%

$$\begin{aligned} P &= \$1 \\ F &= \$2 \\ t &= 20 \text{ years} \\ I &= F - P = \$1 \\ r &= \frac{I}{Pt} = (\$1/\$1)/20 = 0.05 = 5\% \end{aligned}$$

10.3: Three pirates loans and leg replacement

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- In November, Stanley Beard stops by, asks about the weather
  And, oh yes, his brother wants his money back

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- You don't have the money,
  but Stanley Beard loans you the money at the same rate
- In December, Black Beard stops by and demands the money for his brother

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- You don't have the money,
  but Stanley Beard loans you the money at the same rate
- In December, Black Beard stops by and demands the money for his brother
- You don't fancy a wooden leg, so you offer to pay the \$130,
  \$100 plus 3 months simple interest

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- The pirates each charge simple interest, what went wrong?
- You borrowed $110 from Red Beard, so owe him $121
- You borrowed $121 from Stanley Beard, so owe him $133.10

10.3: Compound interest formulas

- The following formula is important enough to memorize:

P = Present value
F = Future value
p = periodic compound interest rate
T = number of periods

$$F = P(1 + p)^T$$

- Same as repeatedly doing simple interest for 1 period

10.3: More compound interest formulas

- These formulas are not worth memorizing, in my opinion

P =	Present value
F =	Future value
APR = r =	annual, nominal, compound interest Rate
n =	Number of periods per year
t =	number of years
APY = r_{eff} =	annual effective Yield (what you actually get)

$$F = P\left(1 + \frac{r}{n}\right)^{(nt)}$$

$$APY = \left(1 + \frac{r}{n}\right)^{(n)} - 1$$

- If $n = \infty$, then we get:

$$F = Pe^{(rt)}$$

10.3: Viewing the pirates as a single bank

- If we view all three brothers as the same lending institution

$$
\begin{aligned}
P &= \$100 \\
p &= 10\% \text{ per month} \\
T &= 3 \text{ months} \\
F &= P(1+p)^T = \$100(1+0.10)^3 = \$100(1.1)^3 \\
&= \$100(1.331) = \$133.10 \\
I &= \$33.10
\end{aligned}
$$

10.3: Annual compounding

- A few investments compound annually

- If you have \$100 in a savings account earning 1% APR, compounded annually,
  How much is it worth in 4 and a half years?

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- If you have \$100 in a savings account earning 1% APR, compounded annually,
  How much is it worth in 4 and a half years? \$104.06, not \$104.58

$$
\begin{aligned}
P &= \$100 \\
p &= 0.01 \\
T &= 4 \text{ not } 4.5 \\
F &= P(1+p)^T = \$100(1.01)^4 = \$104.060401 = \$104.06
\end{aligned}
$$

10.3: Faster compounding

- The three pirates charged 10% per month, compunded monthly
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- Pirates charge 120% APR, because 12 months in the year
- APR is simple, but not all that useful
- APY simply asks "What happens to your dollar after a year?"

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- The three pirates charged 10% per month, compunded monthly
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- Pirates charge 120% APR, because 12 months in the year
- APR is simple, but not all that useful
- APY simply asks "What happens to your dollar after a year?"
- 1% APR compounded annually is 1% APY
- 1% APR compounded monthly is 1.005% APY
- 1% APR compounded every second is 1.005% APY

10.3: APY for consumer loans

- So APR/APY doesn't matter much now for the banks
- But for consumers interest rates are still high:
- 5% APR compounded continuously is 5.127% APY

  Mortgage
- 25% APR compounded continuously is 28.4% APY

  Credit card
- 150% APR compounded bi-monthly is 281.47% APY

  Alternating between two pawn shops