MA111: Contemporary mathematics

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February 8, 2012

Schedule:

- Should have read 10.1-10.3; read 10.6 today (skim 10.4-10.5)
- Should have done HW 10.1-10.3; do 10.6EZ today

Today we will look at borrowing money for several years, 10.6, amortized loans.

10.3: Review of compound interest

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  Value after 5 years?

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- \$100 Savings Account
  earning 2.4% compound interest annually the first two years,
  then 2.1% compound interest annually the next three years,
  Value after 5 years? \$111.60

$$F = \$100(1.024)(1.024)(1.021)(1.021)(1.021) = \$111.60$$

10.3: Review of compound interest

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  earning 2.4% compound interest annually the first two years,
  then you deposit another \$100,
  then 2.4% compound interest annually the next three years,
  Value after 5 years?

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- \$100 Savings Account
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  then you deposit another \$100,
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  Value after 5 years? \$219.97

T	F formula	$=$	F number
Now	\$100.00	=	\$100.00
1st year	(\$100.00)(1.024)	=	\$102.40
2nd year	(\$102.40)(1.024)	=	\$104.86
deposit	\$104.86 + 100	=	\$204.86
3rd year	(\$204.86)(1.024)	=	\$209.78
4th year	(\$209.78)(1.024)	=	\$214.81
5th year	(\$214.81)(1.024)	=	\$219.97

10.3: Review of compound interest

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  earning 2.4% compound interest annually the first two years,
  then you deposit another \$100,
  then 2.4% compound interest annually the next three years,
  Value after 5 years? \$219.97 or \$219.96

- Faster is to think: \$100 was compounded 5 years,
  plus \$100 was compounded 3 years

$$F = \$100(1.024)^5 + \$100(1.024)^3 = \$219.9641731 = \$219.96$$

Installment loans

- What if Black Beard only needed \$20?
  Maybe he'd loan you the rest...
- You owed \$133.10 to Stanley, so after paying \$20 to Mr. Beard, you owe \$113.10, which Black Beard could loan you (to pay back Stanley)
- Next month, maybe Red comes back and only needs \$20, so you owe Black Beard $(\$113.10)(1.1) = \$124.41$ and pay him back \$20, so that is \$104.41 that Red is loaning you.
- If this continued month after month, the amount you owed would go down slowly (not \$20 a month, only \$8.69 the Black-Red month)
- How long does it take to finally pay it off?

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- How long does it take to finally pay it off?
  I get 7 to 8 months, eighth month only costs \$13.72 (not \$20)

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- What kind of loan could we pay off with 10% per month interest?

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- Well, the \$100 for sure, but also the interest that \$100 would have cost us
- The future value is not just \$100, but $\$100(1.1)(1.1) = \$121$
- Total future value is $\$121 + \$110 + \$100 = \$331$
- How much present value is that?
- $P = F/(1+p)^T = \$331/(1.1)^3 = \$248.69$

10.6: Longer installment loans

- What if it was 20 payments? Add them by hand?
- We just use the formula:

$$P = Mq\frac{1-q^T}{1-q} \quad \text{where } q = \frac{1}{1+p}$$

- Here M is the monthly (periodic) payment, and p is the periodic interest rate
- Be careful not to round q (keep 6 to 10 digits)
- For the pirates, $q = \frac{1}{1+0.1} = 1/1.1 = 0.90909090$

$$P = 100(0.90909090)\frac{1-0.90909090^3}{1-0.90909090} = \$248.69$$

10.6: Finding the monthly payment

- What if we needed to borrow \$300 instead. What would the payment be?

$$\$300 = M(0.90909090)\frac{1 - 0.90909090^3}{1 - 0.90909090} = 2.486851942M$$

so

$$M = \$300/2.48685 = \$120.63$$

not much more.