MA111: Contemporary mathematics

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Schedule:

- Read 5.1
- Exam 3 is Friday, March 9th, during class.

Today we will introduce some graphs and Euler circuits.

Each pair of friends has exactly one other friend in common

- 5 people: Alex, Bart, Chad, Dave, Evan
- Some are friends, some are not.
- Alex has the most friends.
- Bart and Chad are friends.
- Dave and Evan are friends.
- Every pair of friends has exactly one other friend in common
- Can you draw a picture of who is friends with whom?

Chapter 5 overview

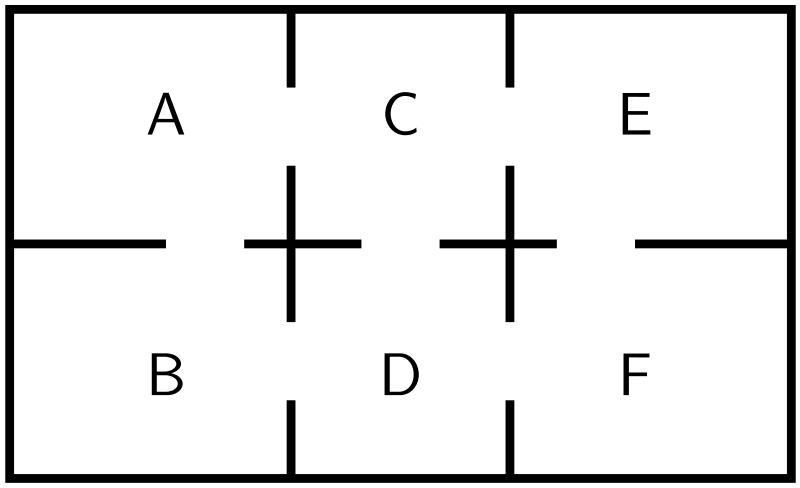
- Graphs have **vertices** and **edges** between the vertices
- Vertices can be virtually anything; a graph only knows which pairs are related and which are not
- We'll be looking at a chain of relations:

$$(A, B) \to (B, C) \to (C, D) \to (D, A) \to (A, C) \to (C, E) \to \dots$$

- If the chain includes all the relations (exactly once, and no non-relations), then we call it an **Euler path**
- Very easy way to tell if they exist
- Reasonable ways to find them when they exist

5.1: Example problems

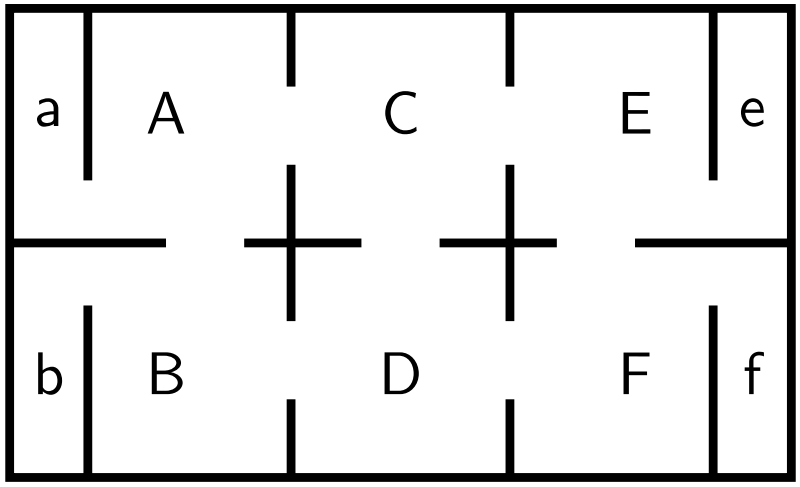
- When I move to a new place, I need to walk through every doorway
- It loses some newness if I go through a doorway more than once
- Can I tour the doors of the house without repeats?



5.1: Example problems

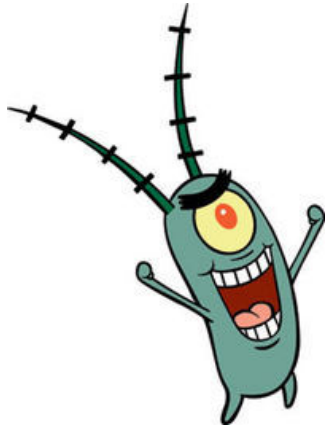
- I forgot the toilets and closets.

- Can I tour the doors of the house without repeats now?



A Halloween Tradition

- Every October, Plankton executes an evil plan.



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- If we only let the people in this room play,
  how many times can you get rid of the toy?

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- The only way to get rid of it is to give it to someone you haven't already given it to
- If we only let the people in this room play,
  how many times can you get rid of the toy?
- Can the toy get stuck before everyone has gotten rid of the toy?

On patrol again

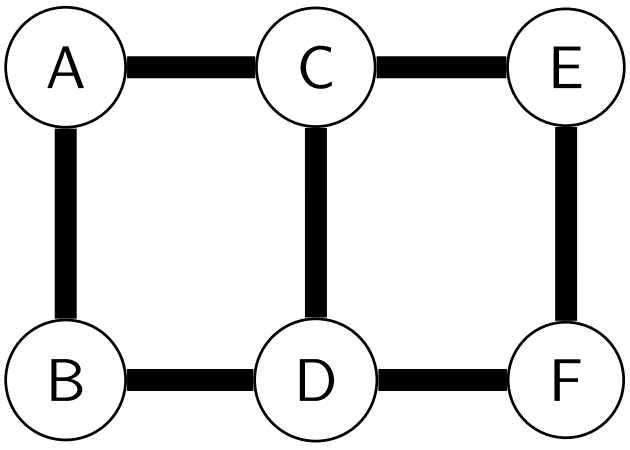
- Can the postal carrier walk every street exactly once
- They should start and end at the Post Office

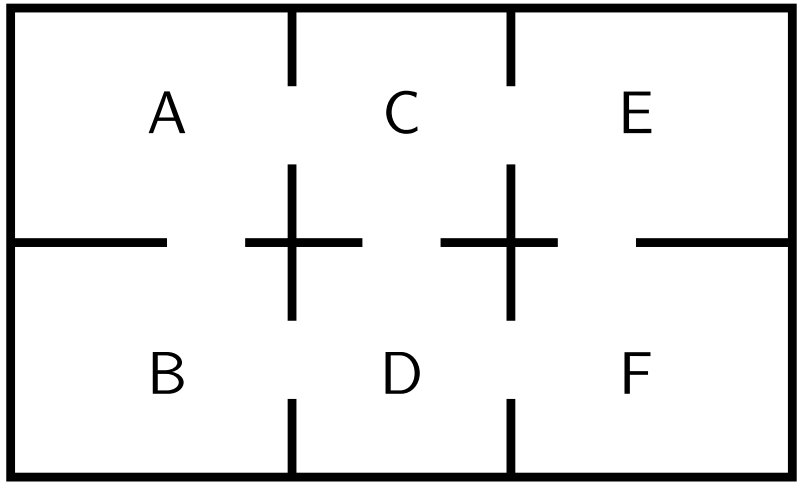
These are all the same question

- Mathematics looks at many different problems
  and finds the common structure
- A strategy to win all similar games
- The house has doors that connect rooms
- The Mr. Blik game has exchanges between people
- The postal carrier has roads between intersections
- The generic has **edges** between **vertices**
  we want to travel over all edges

The house as a graph

- A simpler picture

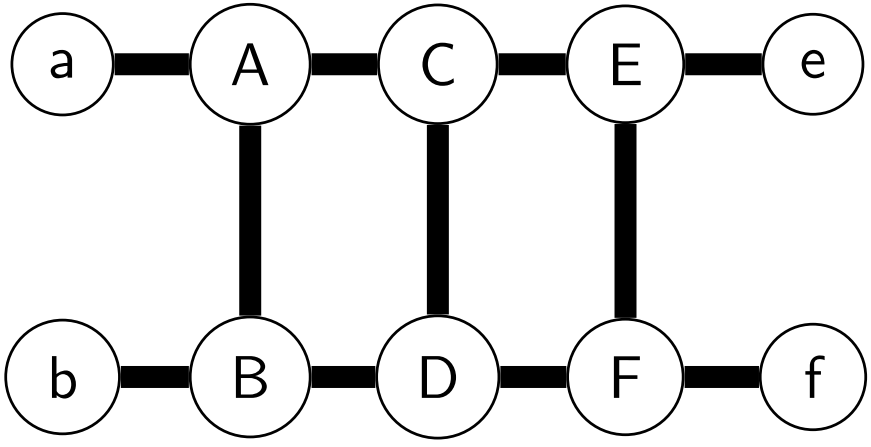


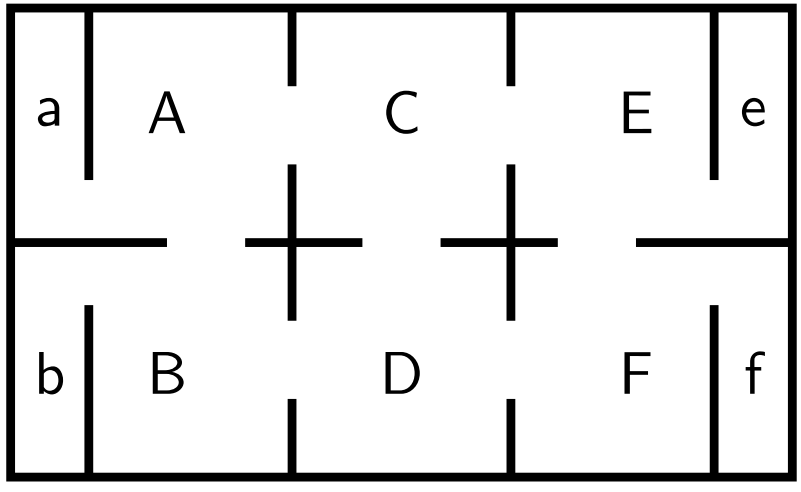


- The connections are the same in both pictures

The house with toilets as a graph

- A simpler picture





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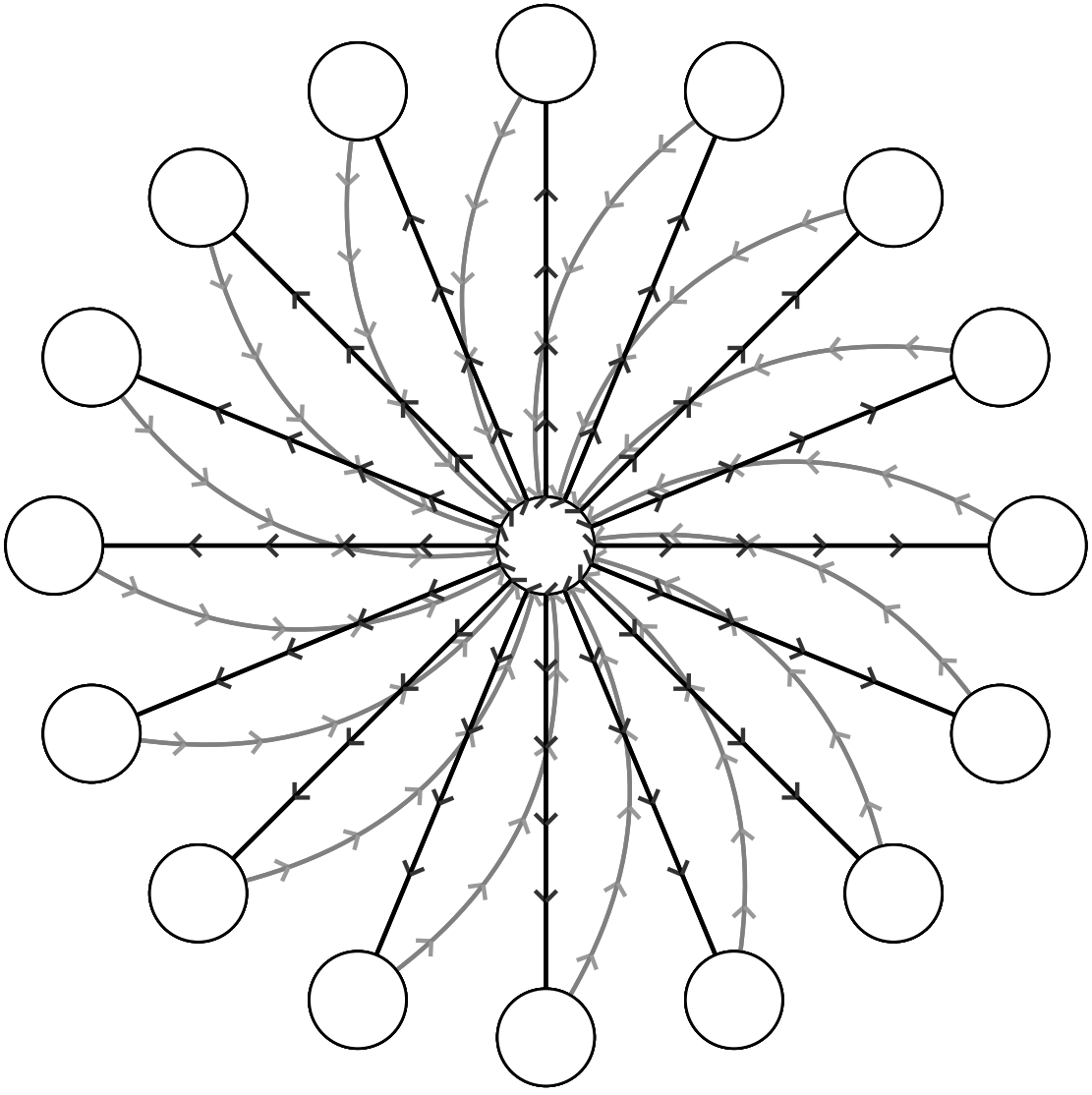
The mean way to play the game

Everyone gives it to Jack, Jack gives it to each person

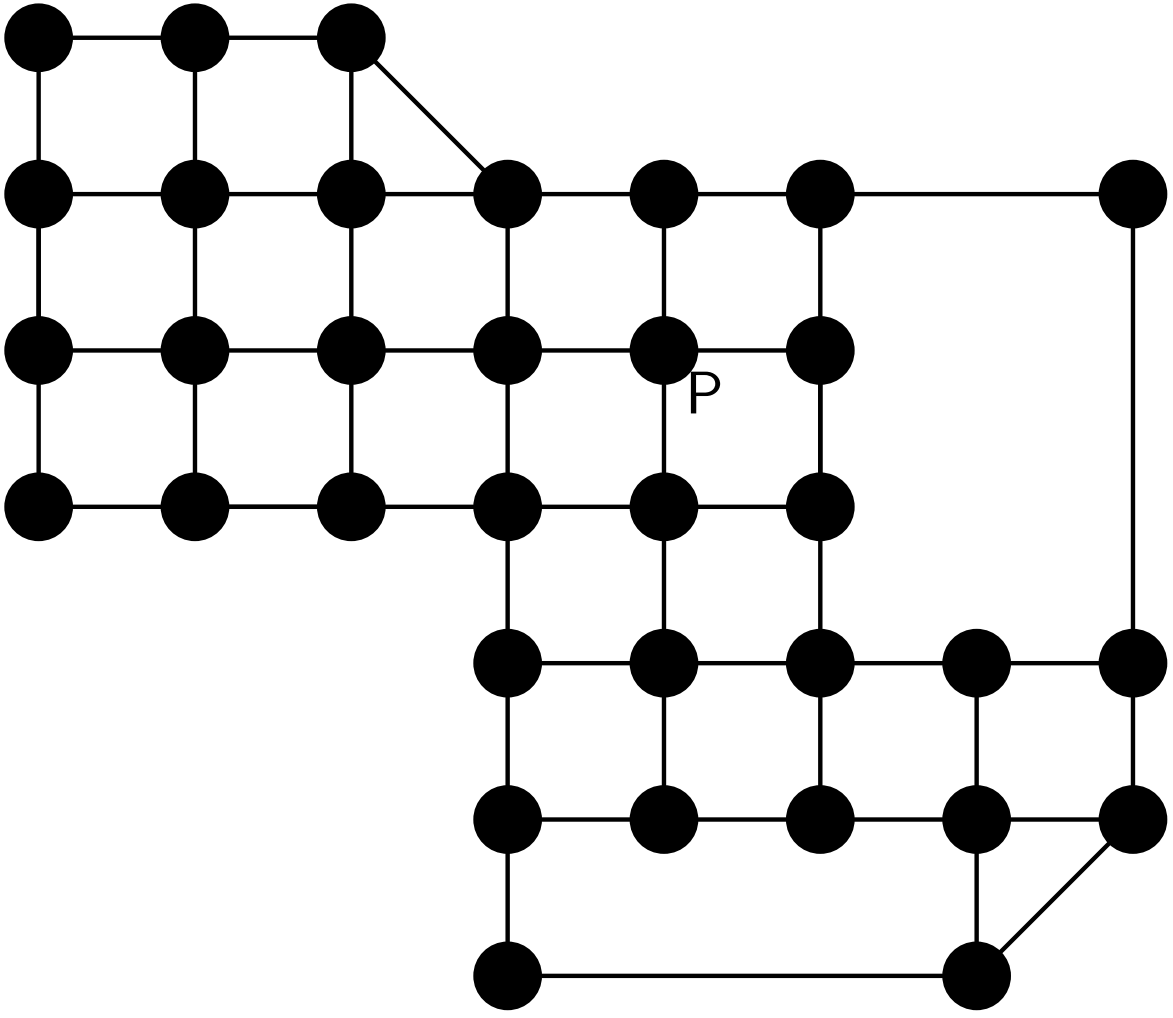


Sharing is caring

- What if you got points for how many times you got rid of the toy?
- "Give it to Jack" is a terrible strategy, since it'll get stuck
  (And Jack will get 70 points!)
- Get in groups of 4 to 5 and figure out how to pass the toy for the maximum number of points
- Be prepared to draw your solution at the board

The postal patrol

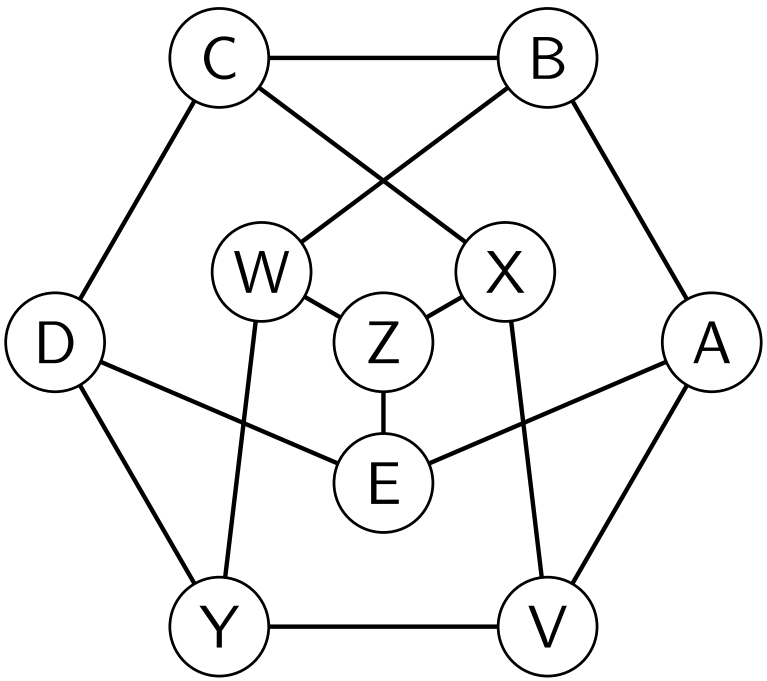
Here is the city as a graph

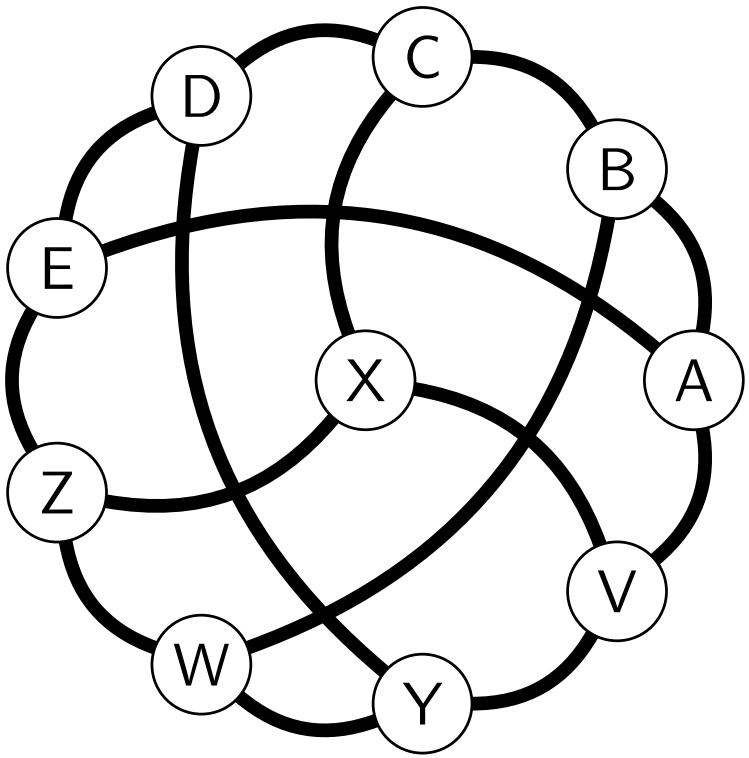


You draw them

- Draw a graph that can be traced
- Draw a graph that obviously can't be traced
- Draw a graph that can't be traced, but that might take a 5 yr old a little time to figure out
- Be prepared to draw at the board!

Can these be traced?





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