

# Practice Exam

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MA111-003  
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## Part I: Vocabulary

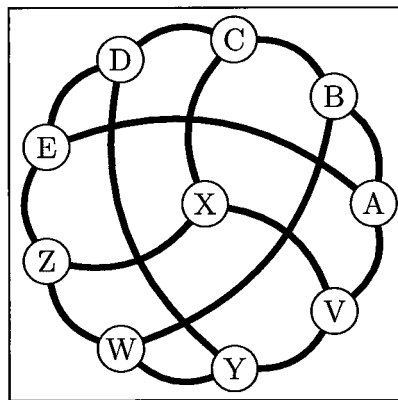
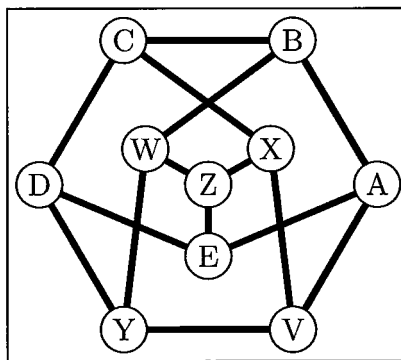
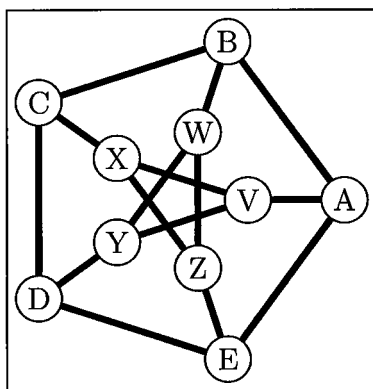
Match the word with its definition:

<u>A</u>	Graph	<u>N</u>	Euler Path
<u>C</u>	Vertex Set	<u>J</u>	Euler Circuit
<u>L</u>	Edge Set	<u>I</u>	Exhaustive Route
<u>K</u>	Degree	<u>H</u>	Eulerization
<u>G</u>	Path	<u>D</u>	Optimal Exhaustive Route
<u>M</u>	Circuit	<u>B</u>	Optimal Eulerization
<u>F</u>	Connected	<u>E</u>	Handshaking Lemma

- (A) A collection of relationships with two parts: a vertex set and an edge set
- (B) An Eulerization of an optimal exhaustive route
- (C) The list of vertices of a graph, or at least a way to tell exactly what the vertices of a graph are
- (D) An exhaustive route of shortest possible length
- (E) The total degree is twice the number of edges
- (F) A graph such that between any two distinct vertices there is a path
- (G) A sequence of edges, each adjacent to the next, that start and stop at different vertices
- (H) A list of the edges to be used more than once in an exhaustive route
- (I) A circuit using all the edges in a graph at least once
- (J) A circuit using all the edges in a graph exactly once
- (K) The number of times a vertex appears in the edge set; the number of edges adjacent to a vertex (where loops count twice)
- (L) The list of edges of a graph, or at least a way to tell how many relationships are between any two vertices
- (M) A sequence of edges, each adjacent to the next, that start and stop at the same vertex
- (N) A path using all the edges in a graph exactly once

## Part II: Definitions and Euler's theorem

Pick one of these graphs and answer the following questions about it.



(a) List the vertices (alphabetically):

A, B, C, D, E, V, W, X, Y, Z (10 vertices)

(b) List the edges (alphabetically):

15 edges, 15 crossed out,

~~AB~~ ~~BA~~ ~~CB~~ ~~DC~~ ~~EA~~ ~~VA~~ ~~WB~~ ~~XC~~ ~~YD~~ ~~ZE~~  
~~AE~~ ~~BC~~ ~~CD~~ ~~DE~~ ~~ED~~ ~~VX~~ ~~WY~~ ~~XV~~ ~~YV~~ ~~ZW~~  
~~AV~~ ~~BW~~ ~~CX~~ ~~DY~~ ~~EZ~~ ~~VY~~ ~~WZ~~ ~~XZ~~ ~~YW~~ ~~ZX~~

(c) What are the degrees of the vertices?

A B C D E V W X Y Z all 3. Total Degree =  $3(10) = 30$   
 $\#edges = 30/2 = 15 \checkmark$

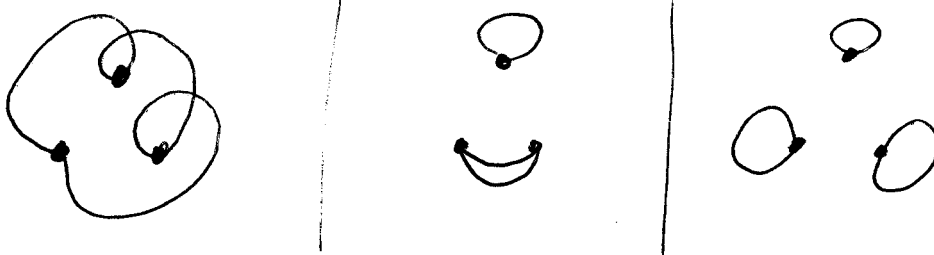
(d) Does this graph have an Euler circuit, an Euler path, both, or neither? Why?

Neither: It has 10 odd vertices.

Euler path requires exactly 2 odd degree vertices  
 Euler cycle requires exactly 0 odd degree vertices

### Part III: Handshaking lemma and graph reconstruction

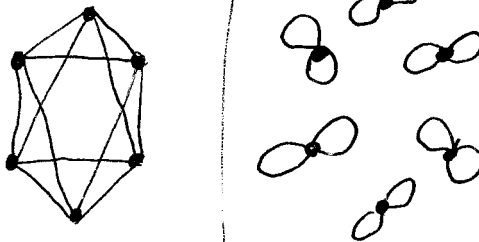
1. Construct a graph with vertices of degree 2, 2, 2 or explain why no such graph exists.



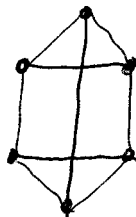
2. Construct a graph with vertices of degree 1, 1, 1, 2, 2 or explain why no such graph exists.

$1+1+1+2+2 = 7$ , so graph has 3.5 edges. What?  
Impossible

3. Construct a graph with vertices of degree 4, 4, 4, 4, 4, 4 (that is six 4s) or explain why no such graph exists.



4. Construct a graph with vertices of degree 3, 3, 3, 3, 3, 3 (that is six 3s) or explain why no such graph exists.

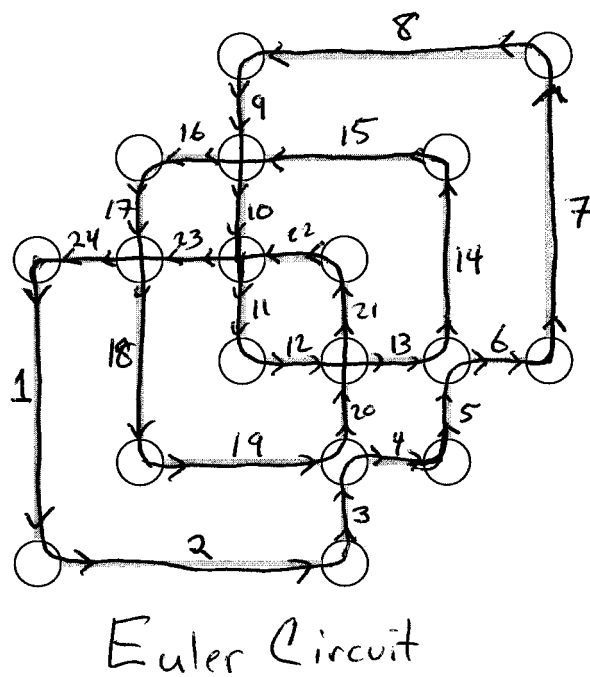
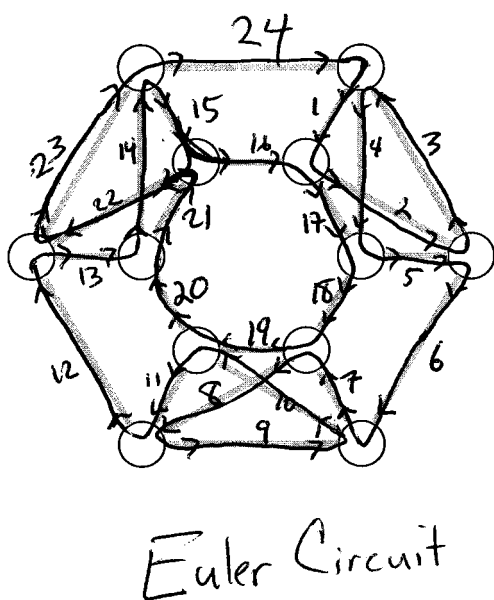
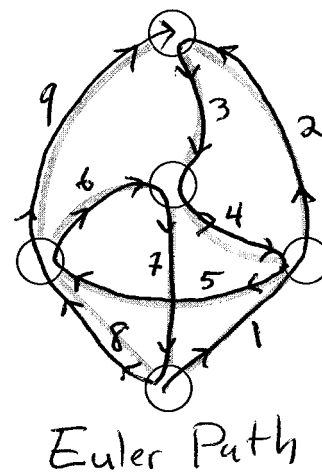
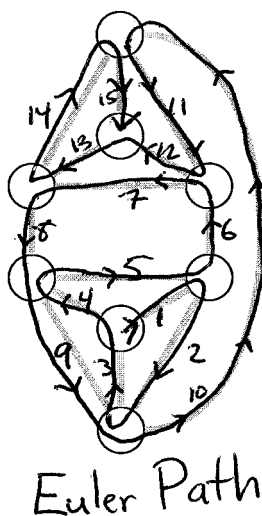
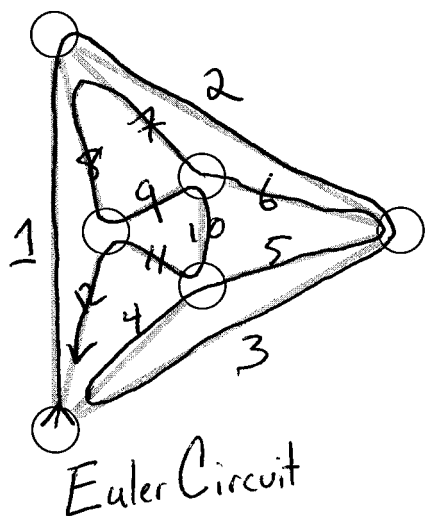


5. Construct a graph with vertices of degree 1, 1, 1, 2, 2, 3, 4 or explain why no such graph exists.



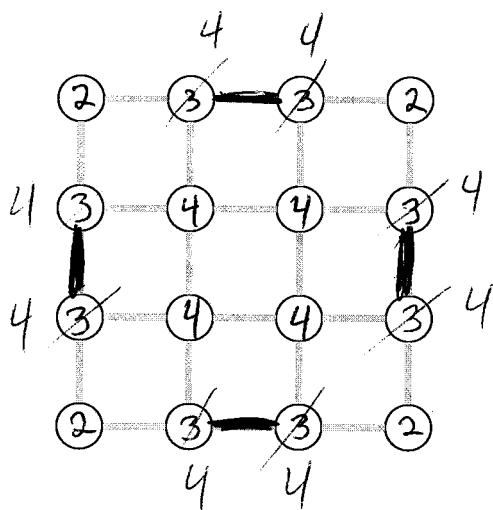
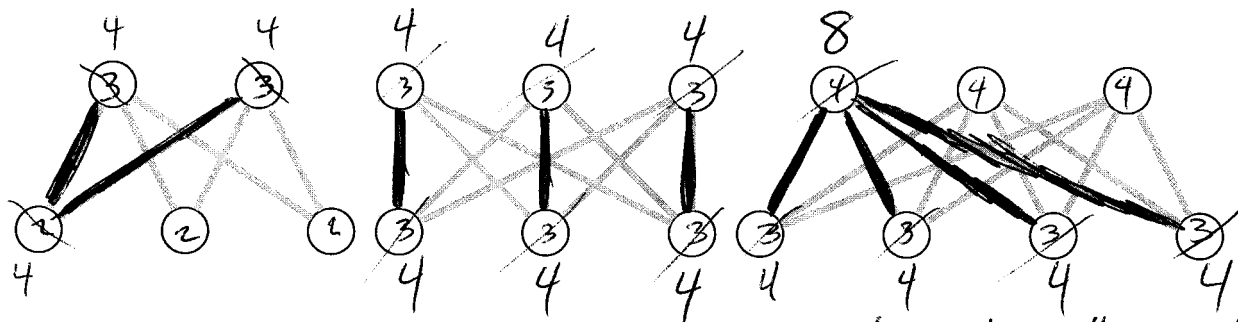
## Part IV: Finding the Euler circuits

1. For each graph label the edges 1, 2, 3, ... in order of an Euler circuit or Euler path.



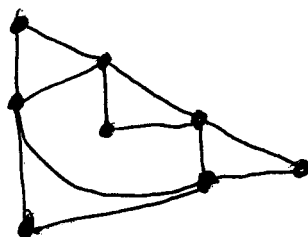
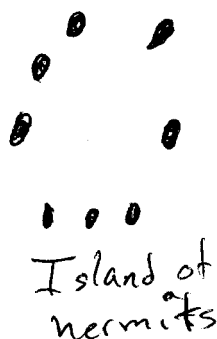
## Part V: No, really, find them!

Label the degrees of each vertex, and then find optimal Eulerizations. Describe the Eulerization by darkening the edges that are repeated (don't add any truly new edges, only repeat old ones, until all the degrees are even). Each of the graphs is connected.

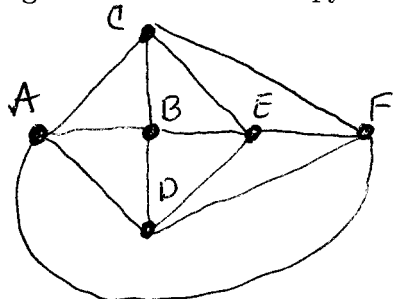


## Part VI: Friends

1. Describe how 8 people can be friends so that every pair of friends has exactly one friend in common. (Draw a graph with 8 vertices so that every edge is a member of exactly one triangle.)

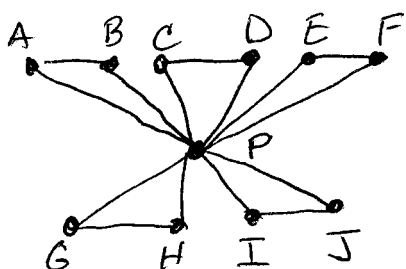


2. Describe how 6 people can be friends so that every pair of friends has exactly two friends in common. (Draw a graph with 6 vertices so that every edge is a member of exactly two triangles. Hint: Stack two pyramids on top of each other.)



AB have C, D in common, but not E, F (only half friends)  
 AC have B, F in common, but not D, E  
 AD have B, F in common, but not C, E  
 AE aren't friends  
 AF have C, D in common, but not B, E  
 The other pairs are similar  
 (The 3D picture is even easier)

3. Describe how 11 people can be friends so that every pair of **people** (not just friends), has exactly one friend in common. (Draw a graph with 11 vertices so that every two vertices has exactly one neighbor in common.)



AB have Mr. P in common  
 AC have Mr. P in common  
 AD have Mr. P in common  
 ...  
 AJ have Mr. P in common  
 ...  
 IJ have Mr. P in common  
 AP have B in common  
 BP have A in common  
 CP have D in common  
 DP have C in common  
 ...