Practice Exam

Name: ____________________
MA111-003
2012-03-07

Part I: Vocabulary

Match the word with its definition:

________	Graph	________	Euler Path
________	Vertex Set	________	Euler Circuit
________	Edge Set	________	Exhaustive Route
________	Degree	________	Eulerization
________	Path	________	Optimal Exhaustive Route
________	Circuit	________	Optimal Eulerization
________	Connected	________	Handshaking Lemma

(A) A collection of relationships with two parts: a vertex set and an edge set

(B) An Eulerization of an optimal exhaustive route

(C) The list of vertices of a graph, or at least a way to tell exactly what the vertices of a graph are

(D) An exhaustive route of shortest possible length

(E) The total degree is twice the number of edges

(F) A graph such that between any two distinct vertices there is a path

(G) A sequence of edges, each adjacent to the next, that start and stop at different vertices

(H) A list of the edges to be used more than once in an exhaustive route

(I) A circuit using all the edges in a graph at least once

(J) A circuit using all the edges in a graph exactly once

(K) The number of times a vertex appears in the edge set; the number of edges adjacent to a vertex (where loops count twice)

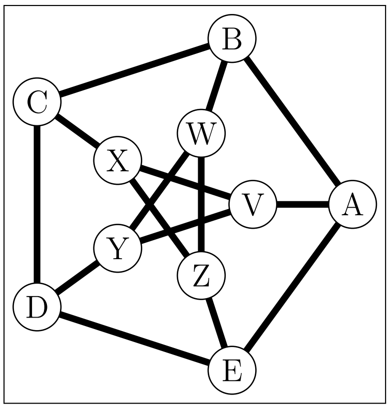
(L) The list of edges of a graph, or at least a way to tell how many relationships are between any two vertices

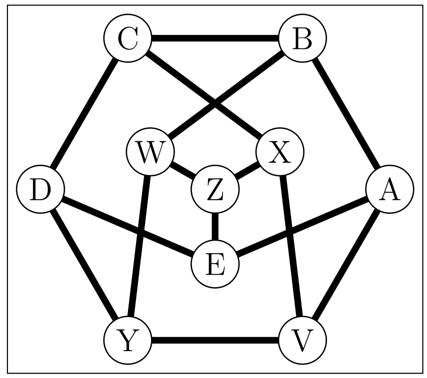
(M) A sequence of edges, each adjacent to the next, that start and stop at the same vertex

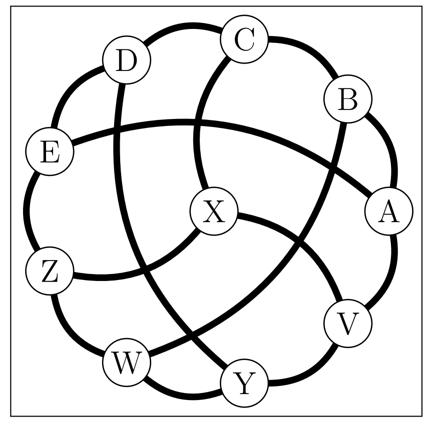
(N) A path using all the edges in a graph exactly once

Part II: Definitions and Euler's theorem

Pick one of these graphs and answer the following questions about it.







(a) List the vertices (alphabetically):

(b) List the edges (alphabetically):

(c) What are the degrees of the vertices?

(d) Does this graph have an Euler circuit, an Euler path, both, or neither? Why?

Part III: Handshaking lemma and graph reconstruction

1. Construct a graph with vertices of degree 2, 2, 2 or explain why no such graph exists.

2. Construct a graph with vertices of degree 1, 1, 1, 2, 2 or explain why no such graph exists.

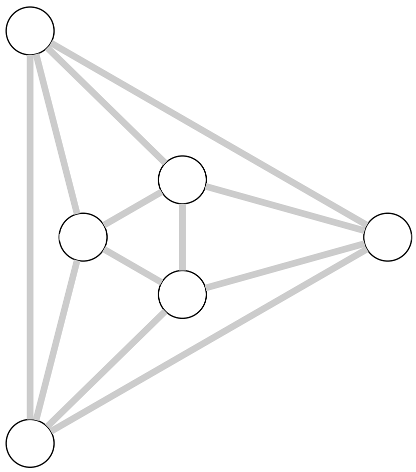
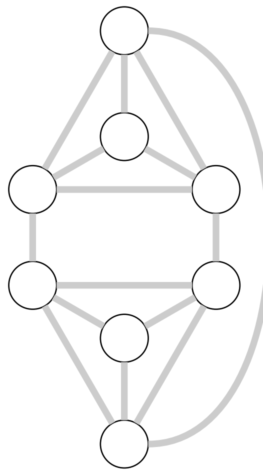
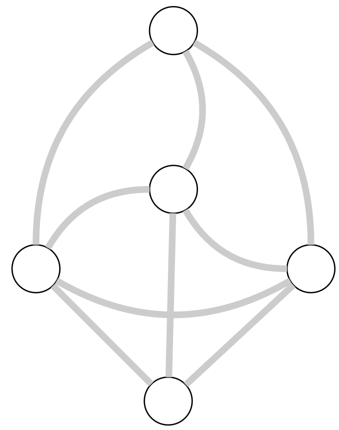
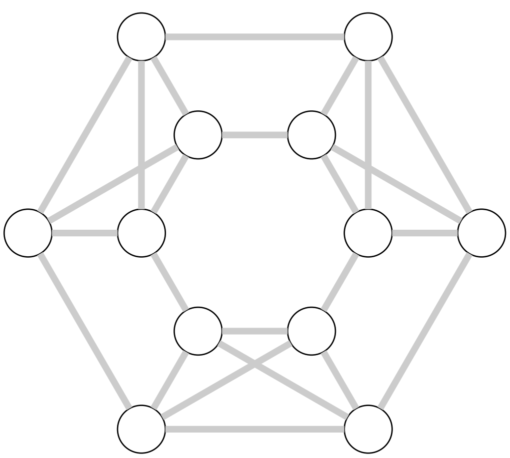
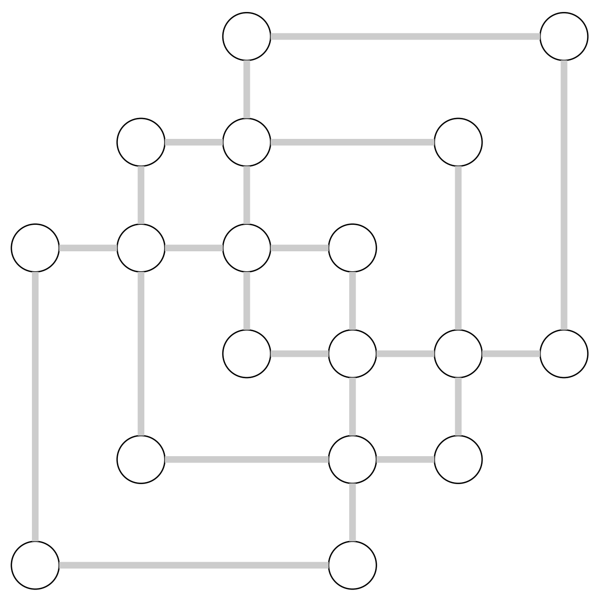
3. Construct a graph with vertices of degree 4, 4, 4, 4, 4, 4 (that is six 4s) or explain why no such graph exists.

4. Construct a graph with vertices of degree 3, 3, 3, 3, 3, 3 (that is six 3s) or explain why no such graph exists.

5. Construct a graph with vertices of degree 1, 1, 1, 2, 2, 3, 4 or explain why no such graph exists.

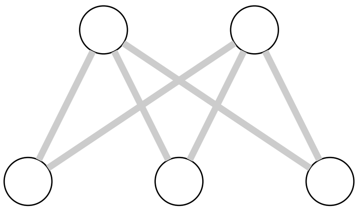
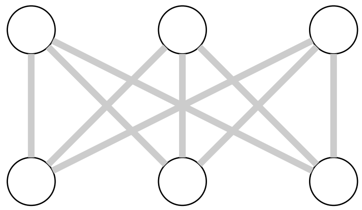
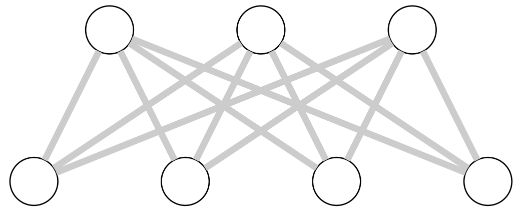
Part IV: Finding the Euler circuits

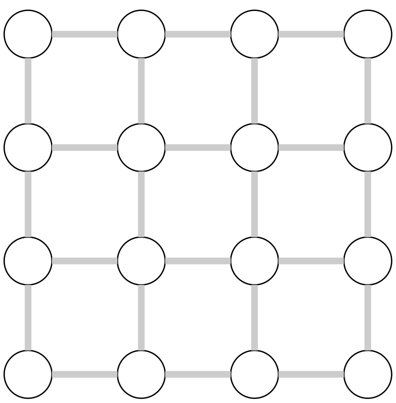
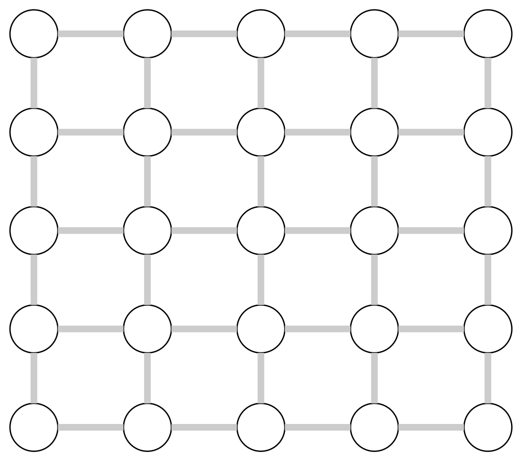
1. For each graph label the edges $1, 2, 3, \ldots$ in order of an Euler circuit or Euler path.

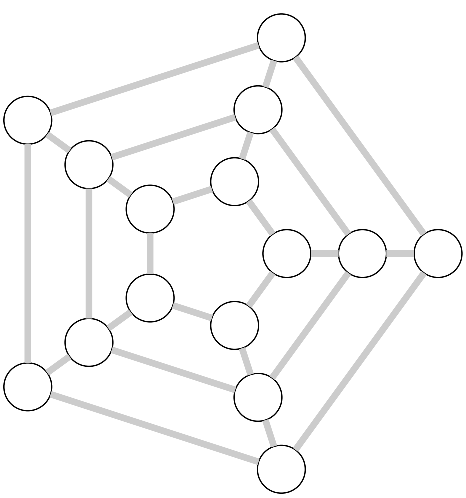
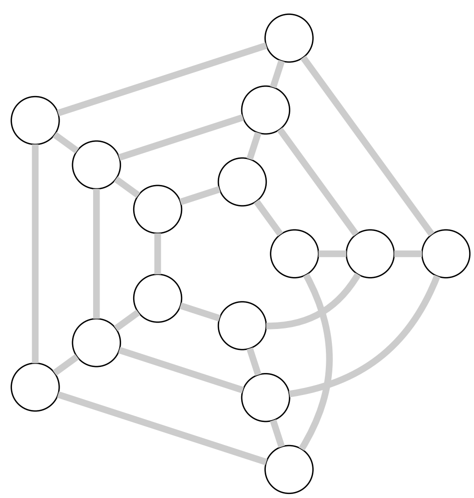






Part V: No, really, find them!

Label the degrees of each vertex, and then find optimal Eulerizations. Describe the Eulerization by darkening the edges that are repeated (don't add any truly new edges, only repeat old ones, until all the degrees are even). Each of the graphs is connected.

Part VI: Friends

1. Describe how 8 people can be friends so that every pair of friends has exactly one friend in common. (Draw a graph with 8 vertices so that every edge is a member of exactly one triangle.)

2. Describe how 6 people can be friends so that every pair of friends has exactly two friends in common. (Draw a graph with 6 vertices so that every edge is a member of exactly two triangles. Hint: Stack two pyramids on top of each other.)

3. Describe how 11 people can be friends so that every pair of **people** (not just friends), has exactly one friend in common. (Draw a graph with 11 vertices so that every two vertices has exactly one neighbor in common.)