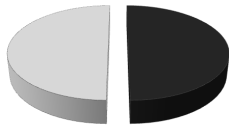
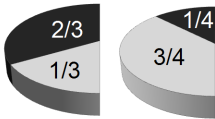


1. Abby and Ziva jointly buy the half chocolate-half vanilla cake shown for \$24. Suppose that Ziva feels that 80% of the value of the cake is in the chocolate half, and 20% of the value is in the vanilla half. Find the dollar value to Ziva for each of the following pieces:



(a) The vanilla half of the cake.



(b) The chocolate half of the cake.

(c) A slice which is $\frac{1}{3}$ of the vanilla and $\frac{2}{3}$ of the chocolate.

(d) A slice is which $\frac{1}{4}$ of the chocolate and $\frac{3}{4}$ of the vanilla.

(e) How could Ziva slice the cake into two fair shares?

2. Three players (Richard, Kahlan and Zed) are sharing a cake. Suppose that the cake is divided into three slices (s_1, s_2, s_3). The following table gives the value of s_1 and s_2 to each of the players.

	s_1	s_2	s_3
Richard	30%	34%	
Kahlan	28%	36%	
Zed	30%	$33\frac{1}{3}\%$	

(a) Fill in the missing percents for each player.

(b) Which of the three slices are fair shares to Richard?

(c) Which of the three slices are fair shares to Kahlan?

(d) Which of the three slices are fair shares to Zed?

(e) Find a fair division of the cake using s_1, s_2 and s_3 as fair shares. If this is not possible, explain why not.

3. Three players (Shawn, Gus and Juliet) are sharing a cake. Suppose that the cake is divided into three slices (s_1, s_2, s_3). The following table gives the value of each slice in the eyes of each of the players.

	s_1	s_2	s_3
Shawn	\$6.00	\$7.00	\$8.00
Gus	\$6.00	\$6.50	\$5.50
Juliet	\$8.00	\$6.25	\$6.75

(a) Which of the three slices are fair shares to Shawn?

(b) Which of the three slices are fair shares to Gus?

(c) Which of the three slices are fair shares to Juliet?

(d) Find a fair division of the cake using s_1, s_2 and s_3 .

Shawn: _____

Gus: _____

Juliet: _____

4. Four players (Ben, Johnny, Reed and Sue) are sharing a cake. Suppose that the cake is divided into four slices s_1, s_2, s_3 and s_4 . Below is a partial table of the values that each player assigned to each slice.

	s_1	s_2	s_3	s_4
Ben	\$3.60			\$3.50
Johnny	\$1.00			
Reed			\$5.00	
Sue	\$6.00	\$3.00	\$5.00	\$4.00

- (a) To Ben, the entire cake is worth \$15.00. If s_2 and s_3 are equal values to Ben, fill in the missing values for Ben. Determine which of the four slices are fair shares to Ben.
- (b) To Johnny, s_2 is worth twice as much as s_1 , s_3 is worth three times as much as s_1 , and s_4 is worth four times as much as s_1 . Fill in the missing values for Johnny. Determine which of the four slices are fair shares to Johnny.
- (c) To Reed, s_1, s_2 and s_4 have equal value, and the entire cake is worth \$11.00. Fill in the missing values for Reed. Determine which of the four slices are fair shares to Reed.

5. Danny and Sandy are planning on splitting the pizza shown in Figure 1 using the divider-chooser method. The pizza is $\frac{1}{3}$ pepperoni, $\frac{1}{3}$ sausage, and $\frac{1}{3}$ cheese. Danny likes pepperoni and sausage equally well, and he likes cheese twice as much as sausage or pepperoni. Sandy is a vegetarian, so she places 100% of the value in the cheese third. For all questions below, assume that Danny and Sandy know nothing about the other's likes and dislikes.

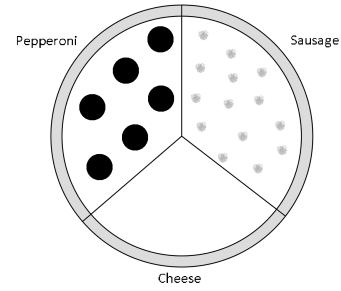
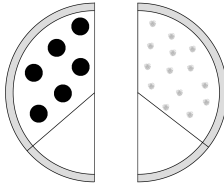
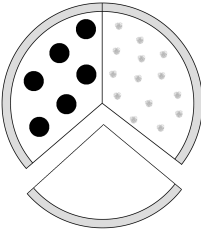


Figure 1

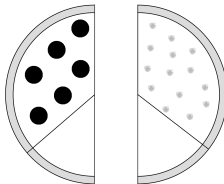
- (a) Is the cut shown below a cut that Danny might have made as the divider? If so, describe the share that Sandy would choose.



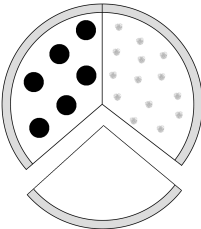
- (b) Is the cut shown below a cut that Danny might have made as the divider? If so, describe the share that Sandy would choose.



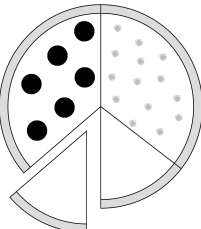
- (c) Is the cut shown below a cut that Sandy might have made as the divider? If so, describe the share that Danny would choose.



- (d) Is the cut shown below a cut that Sandy might have made as the divider? If so, describe the share that Danny would choose.



- (e) Is the cut shown below a cut that Sandy might have made as the divider? If so, describe the share that Danny would choose.



6. Three partners are dividing a plot of land among themselves using the lone-divider method. After the divider D divides the land into three shares s_1, s_2 , and s_3 , the choosers C_1 and C_2 submit their bids for these shares.

- (a) Suppose the chooser's bids are $C_1 : \{s_2\}$ and $C_2 : \{s_1, s_3\}$. Describe two different fair divisions of the land.

Fair Division 1:

C_1 : _____

C_2 : _____

D : _____

Fair Division 2:

C_1 : _____

C_2 : _____

D : _____

- (b) Suppose the chooser's bids are $C_1 : \{s_1, s_2\}$ and $C_2 : \{s_1, s_2\}$. Describe two different fair divisions of the land.

Fair Division 1:

C_1 : _____

C_2 : _____

D : _____

Fair Division 2:

C_1 : _____

C_2 : _____

D : _____

7. Four partners are dividing a plot of land among themselves using the lone-divider method. After the divider D divides the land into four shares s_1, s_2, s_3 , and s_4 , the choosers C_1, C_2 , and C_3 submit their bids for these shares.

- (a) Suppose that the chooser's bids are $C_1 : \{s_2\}$, $C_2 : \{s_1, s_3\}$, $C_3 : \{s_2, s_3\}$. Find a fair division of the land.

Fair Division:

C_1 : _____

C_2 : _____

C_3 : _____

D : _____

- (b) Explain why the fair division you found in part (a) is the only fair division in that situation.

8. Four players (Alex, Bailey, Christina, Derek) are dividing a pizza worth \$18 among themselves using the lone-divider method. The divider divides the pizza into four shares s_1, s_2, s_3 , and s_4 . The following table shows some of the values of the four shares in the eyes of each player.

	s_1	s_2	s_3	s_4
Alex	\$5.00	\$5.00	\$3.50	
Bailey		\$4.50		
Christina	\$4.80	\$4.20	\$4.00	
Derek	\$4.00	\$3.75	\$4.25	

- (a) Fill in the missing values in the table above.
- (b) Who was the divider?
- (c) How much must a share be worth for it to be a fair share in this situation?
- (d) Determine each chooser's bid.
- (e) Find a fair division of the pizza.

Fair Division:

Alex: _____

Bailey: _____

Christina: _____

Derek: _____