MA111: Contemporary mathematics

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April 20, 2012

Schedule:

- Guest lecture on Wednesday: the Ham-Sandwich theorem
- Homework (written) due Today Friday April 20th and Friday April 27th.
- Written project due Today Friday April 20th:

  Two well-written homework answers (page 119-120, Chapter 3):

  1. Answer one of #77, #78, #79, #80
  2. Answer one of #81, #82, #83

3.6: Today we will study how to divide the undivisible.

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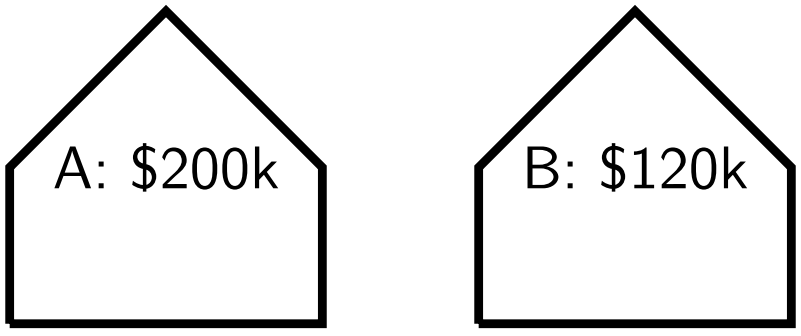
- Flip a coin; winner gets the house, but pays the loser the "fair market value" of the house

Amelia and Bill have their own ideas

- Bill hates the house. The fair market value is $150k, but he wants it sold now, and would take $120k today.

- Amelia loves the house and has spent about $50k of loving attention improving the house in little ways only she can see. She wouldn't sell it for less than $200k.



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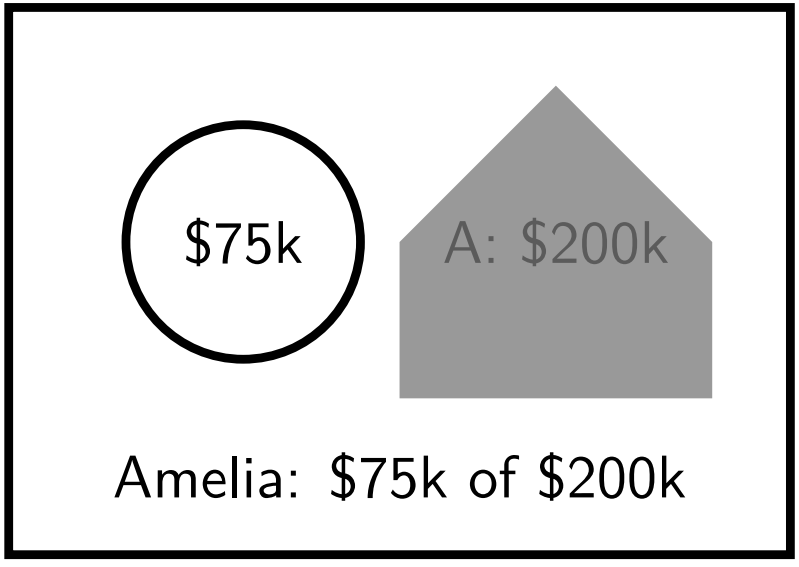

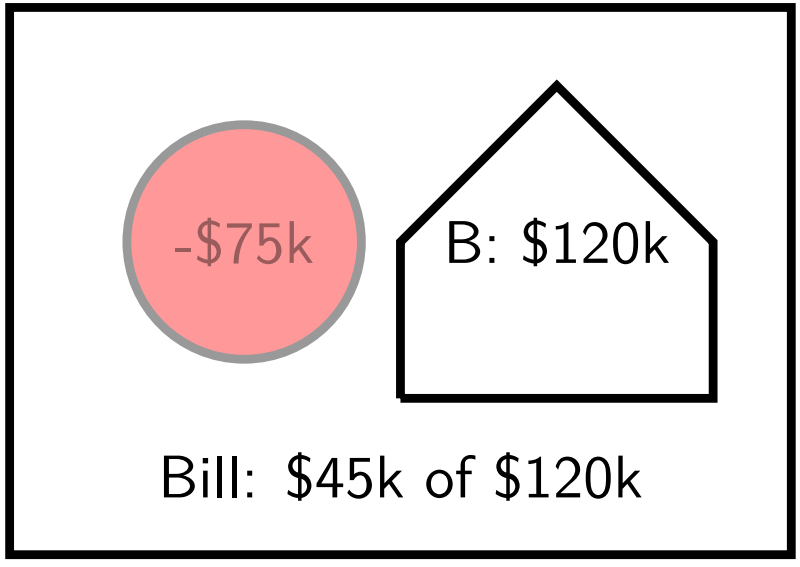

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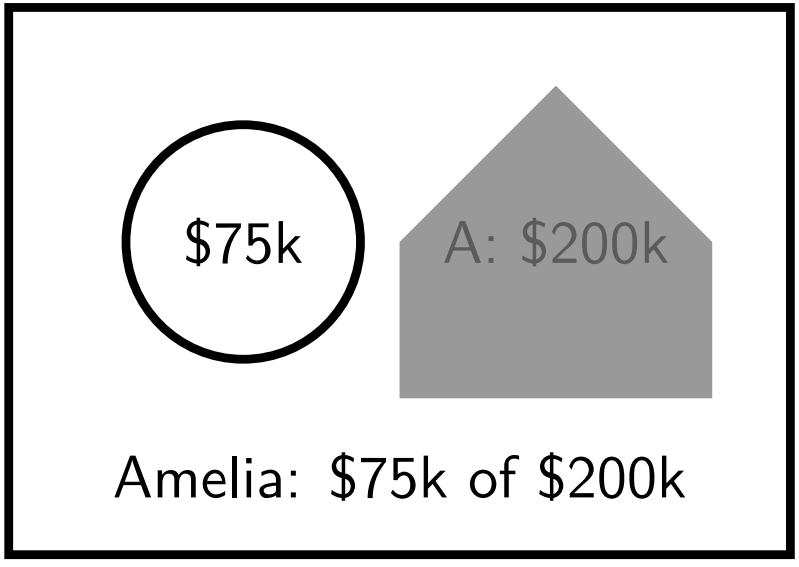
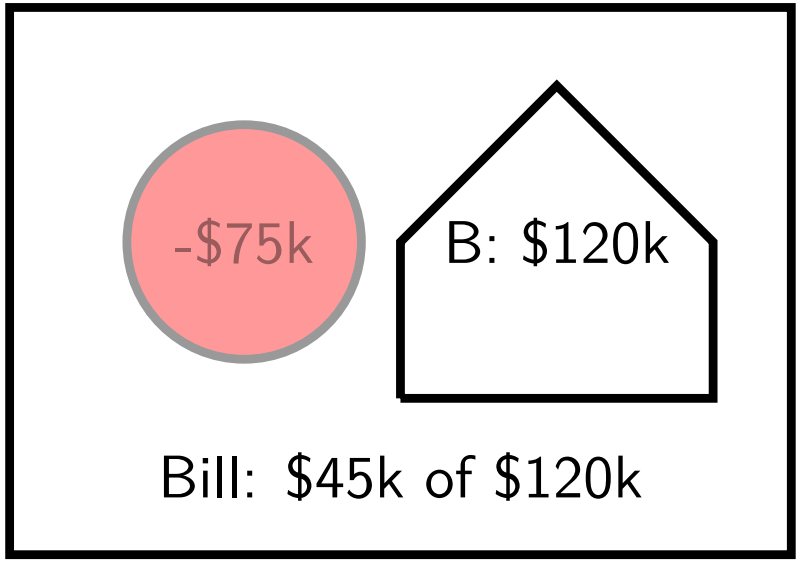
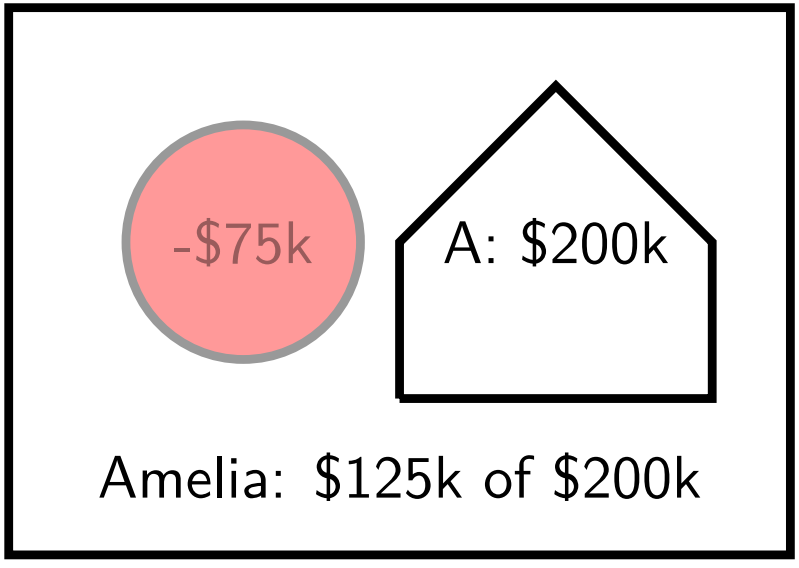


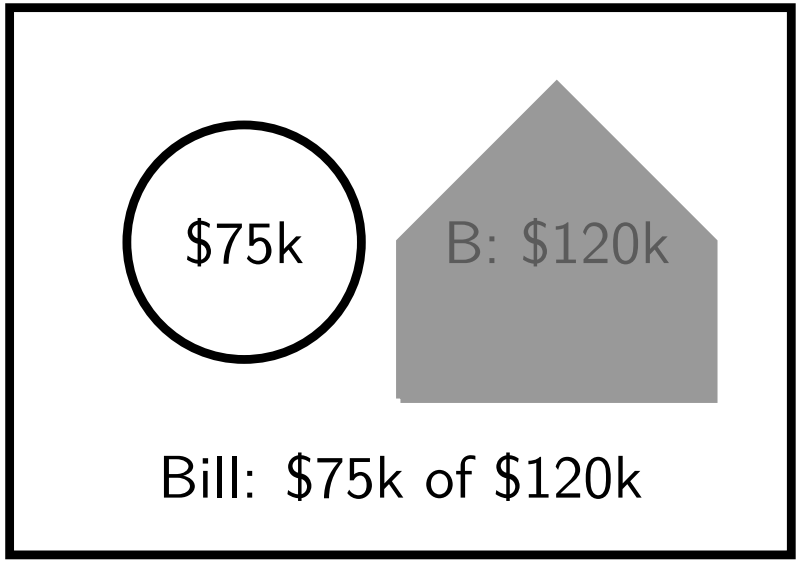


- Neither thinks this is fair! Both got only 37.5% of the house!
  25% of the house was lost and everyone is mad!

Who should really get the house?

- Amelia wants the house more, Bill wants the money more.
- If they just trade, things are much better:





- They are both happy: Amelia gets 62.5%, Bill gets 62.5%.
  An extra 25% of the house was created from nothing!

Auctions and compensation agreements

- Several people jointly own an item
- Submit sealed bids
- One of the bidders is given the item in exchange for their bid
- The bid is divided amongst the others somehow
- So two questions:
  - Which bid gets the item?
  - How to split the money?

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- Bill got less value, but more percentage.
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