MA162: Finite mathematics

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January 11, 2012

Schedule:

- HW 0.1 is due Friday, Jan 13th, 2012.
- HW 0.2 is due Tuesday, Jan 17th, 2012.
- HW 1.1-1.4 are due Friday, Jan 20th, 2012.
- Exam 1 is Monday, Feb 6th, 5:00pm-7:00pm in CB106 and CB118.

Today we will cover: 1.3 linear functions; linear depreciation; cost, revenue, profit

Ch 1.3: Example 1: Linear depreciation

- In accounting, you keep track of assets (goods)
- But assets are also tax liabilities (bads)
- Old assets are like so whatever and are worth less
- For example:

  A printing machine is currently worth \$100,000, but will be depreciated over five years to its scrap value of \$30,000.

  How much is the machine worth after two years?

Ch 1.3: Example 1: Linear depreciation

- For example:

  A printing machine is currently worth \$100,000, but will be depreciated over five years to its scrap value of \$30,000.

  How much is the machine worth after two years?

- Over five years, it loses \$70k of value
- Each year it loses $\$70k/5 = \$14k$ of value
- After two years, it loses $\$14k * 2 = \$28k$
- It is worth \$72k by the end of the second year

Ch 1.3: Example 1: Linear depreciation

- This is just **slope**:
- $(x = 0, y = \$100k)$ and $(x = 5, y = \$30k)$ are two points on the graph
- The slope is

$$\frac{100 - 30}{0 - 5} = -14 \text{ thousand dollars per year}$$

- The bunny hops down \$14k every year.
- The **y-intercept** was the original \$100k starting value

Ch 1.3: Example 2: Cost, Revenue, Profit

- To get into the lucrative cell-phone washing business, you just need about \$5 in polishing rags and a winning smile
- However, each wash requires about \$0.05 in disinfectant
- If you charge \$0.25 per wash, how much money will you make if you wash 10 phones? 25 phones? 100?

Ch 1.3: Example 2: Cost, Revenue, Profit

- Well your costs are easy: \$5 plus \$0.05 per wash

$$C(x) = 5 + 0.05x$$

- Your revenue is easy: \$0.25 per wash

$$R(x) = 0.25x$$

- So profit is easy, you start \$5 in the hole, and make \$0.20 per wash

$$P(x) = -5 + 0.20x$$

Ch 1.3: Example 2: Cost, Revenue, Profit

- At 10 washes, you've made \$2.50 but spent \$5.50, so you are \$3 in debt
- At 25 washes, you've made \$6.25 but spent \$6.25, so you just broke even
- At 100 washes, you've made \$25 but spent \$10, so you are \$15 ahead

Ch 1.3: Example 2: Cost, Revenue, Profit

- **Marginal cost** is \$0.05 per wash
- **Marginal profit** is \$0.20 per wash
- **Fixed cost** is \$5
- **Break-even production** is 25 washes

Ch 1.3: Did we understand it?

- Fixed and marginal cost
- 20 cost \$200, 25 cost \$220, how much do 30 cost?

(Left) \$300

(Right) \$240

(Both) \$225

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- Discuss with your neighbors, because you'll explain it to us next

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- 20 cost \$200, 25 cost \$220, how much do 30 cost?

(Left) \$300

(Right) \$240

(Both) \$225

- Discuss with your neighbors, because you'll explain it to us next
- Now explain it to us, especially someone who changed their mind.

Ch 1.3: Did we understand it?

- 20 cost \$200, 25 cost \$220, how much do 30 cost?

(Left) \$300 – This assumes each one costs \$10,
but then 25 should have costed \$250

(Right) \$240 – 5 more costed \$20 more,
so another 5 costs another \$20

(Both) 5 more costs \$5 more? Life isn't that simple

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- So **Marginal cost** is \$20 per 5, or \$4 each

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- So **Marginal cost** is \$20 per 5, or \$4 each
- So **fixed cost** is \$120

Ch 1.3: Do we understand it now?

- 50 cost \$500, 100 cost \$700, how much do 75 cost?

(Left) \$750

(Right) \$900

(Both) \$600

Ch 1.3: Do we understand it now?

- 50 cost \$500, 100 cost \$700, how much do 75 cost?

(Left) \$750

(Right) \$900

(Both) \$600

- 50 more cost \$200 more, so 25 more only costs \$100 more
  (Both) \$600

Ch 1.3: Do we understand it now?

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(Left) \$750

(Right) \$900

(Both) \$600

- 50 more cost \$200 more, so 25 more only costs \$100 more
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- **Marginal cost** is \$4 each

Ch 1.3: Do we understand it now?

- 50 cost \$500, 100 cost \$700, how much do 75 cost?

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(Right) \$900

(Both) \$600

- 50 more cost \$200 more, so 25 more only costs \$100 more
  (Both) \$600

- **Marginal cost** is \$4 each

- **Fixed cost** is \$300,
  since \$4 each for 50 is only \$200, not \$500

Ch 1.4: Intersecting lines: Examples 2-5

- The break-even point is when the **revenue equals the cost**
- $R(x) = C(x)$
- To solve $0.25x = 5 + 0.05x$, move the xs over to get

$$0.20x = 5 \qquad x = 5/0.20 = 25$$

- A pessimistic phrasing is when the **profit is zero**
- $P(x) = 0$
- To solve $-5 + 0.20x = 0$, move the 5 over to get

$$0.20x = 5 \qquad x = 5/0.20 = 25$$

Ch 1.3: Example 3: Demand function

- All else being equal, more people are willing to buy at a lower price
- Hopefully everyone took a syllabus last week
- Not very many people would take it if I charged \$1 per syllabus
- If 150 syllabi are taken at \$0 and none are taken at \$1, about how many would be taken at \$0.02?

Ch 1.3: Example 3: Demand function

- With a **linear demand** model, this is easy:
- Every extra dollar I charge, I lose 150 customers
- If I only charge two extra pennies, I lose $150 * 0.02 = 3$ customers
- 147 pieces of paper should still circulate
- Real demand **curves** are not linear, but if the change in price is small enough, then they are like lines (remember MA123; curves look like lines close up; the derivative)

Ch 1.3: Example 4: Supply function

- All else being equal, more are willing to sell if the price is higher
- If you heard Ovid's ran out of drinks and was paying $20 per bottle of coke, some of you might leave class to make some money
- If no one is willing to supply coke for free, but 150 are willing to supply at $100 per bottle, how many would be willing at $20 per bottle?

Ch 1.3: Example 4: Supply function

- All else being equal, more are willing to sell if the price is higher
- If you heard Ovid's ran out of drinks and was paying \$20 per bottle of coke, some of you might leave class to make some money
- If no one is willing to supply coke for free, but 150 are willing to supply at \$100 per bottle, how many would be willing at \$20 per bottle?
- By increasing the price \$100, we got 150 more sellers
- If we only increased the price a fifth of that, \$20, we would only get 30 more sellers

Ch 1.4: Example 6-7: Market equilibrium

- In a rational, free market, the demand (number of items bought) equals the supply (number of items sold)
- On the exam, a problem like this requires you to:
  - find the supply equation
  - find the demand equation
  - set them equal to each other
  - solve for the **equilibrium quantity**
  - substitute back in for the **equilibrium price** (or vice versa)

Ch 1.3 and 1.4 summary

- Concentrate on how the slope answers most of these questions with bunny hops
- There are also **tax** and **temperature** questions in the textbook
- The homework and exams will use words like: **linear depreciation, cost function, revenue function, profit function, fixed costs, variable costs, supply equation, demand equation, market equilibrium**
- Homework is due Friday, 1.1-1.4
- I am heading to the mathskeller now