MA162: Finite mathematics

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February 15, 2012

SCHEDULE:

- HW 2.6, 3.1 due Friday Feb 17, 2012
- HW 3.2, 3.3 due Friday Feb 24, 2012
- HW 4.1 due Friday Mar 2, 2012
- Exam 2 is Monday, Mar 5, 2012 in CB106 and CB118

Today we will cover 3.1: graphing inequalities

Exam 2: Overview

- 22% Ch. 2, Matrix arithmetic

- 33% Ch. 3, Linear optimization with 2 variables

  1. Graphing linear inequalities

  2. Setting up linear programming problems

  3. Method of corners to find optimum values of linear objectives

- 45% Ch. 4, Linear optimization with millions of variables

  1. Slack variables give us flexibility in RREF

  2. Some RREFs are better (business decisions) than others

  3. Simplex algorithm to find the best one using row ops

  4. Accountants and entrepreneurs are two sides of the same coin

Chapter 3 and 4: Example problem

- Mr. Marjoram decides to use his machines to make that money

- Each of his products earns him some profit,
  but requires manufacturing time

	Panda	Dog	Bird	Rented
Sewing	15 min per	20 min per	25 min per	1100 minutes
Stuff	30 min per	35 min per	25 min per	1400 minutes
Trim	12 min per	8 min per	5 min per	350 minutes
Profit	$10 per	$15 per	$12 per	

- How many of each product should he make in order to maximize
  profit using **at most** the available time?

- Work on it in groups.

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- How many of each product should he make in order to maximize profit using **at most** the available time?

- Work on it in groups. Can you beat $636?

3.1: Inequalities

- Xylophones cost $200 each and Yukuleles cost $100 each

- Your need instruments for your new band Glük-N-Spiel

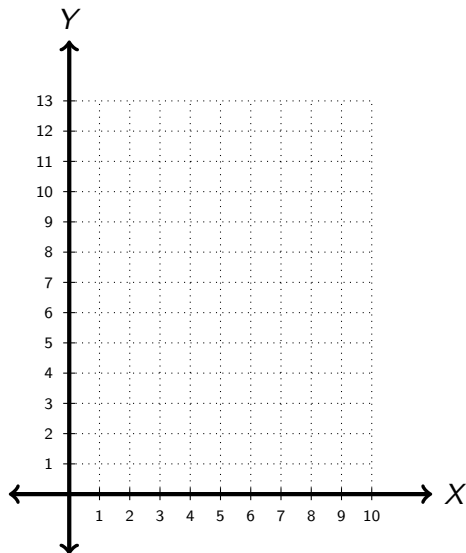- Your insane and rich uncle only gave you a budget of $1000

- What are your options?

$$200x + 100y = 1000$$

3.1: Inequalities

- Xylophones cost $200 each and Yukuleles cost $100 each

- Your need instruments for your new band Glük-N-Spiel

- Your insane and rich uncle only gave you a budget of $1000

- What are your options? Don't have to spend it all!

$$200x + 100y \leq 1000$$

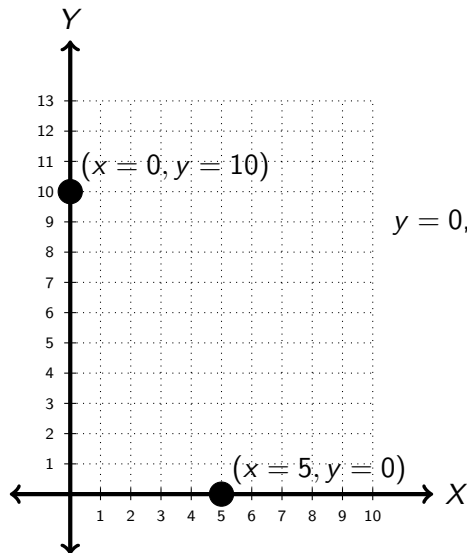## 3.1: Graphing inequalities

$$200x + 100y = 1000$$

3.1: Graphing inequalities
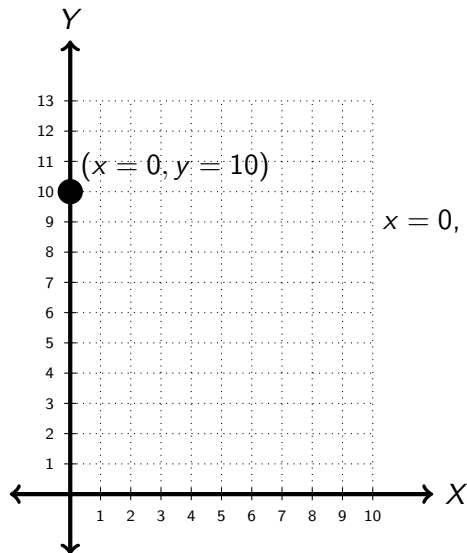
$$200x + 100y = 1000$$
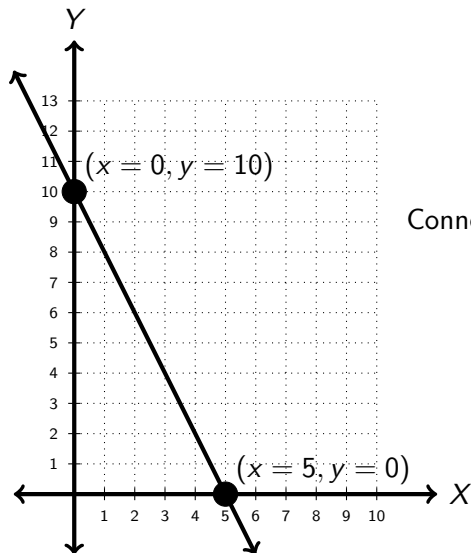
$x = 0$, $100y = 1000$, $y = 10$

3.1: Graphing inequalities

$$200x + 100y = 1000$$
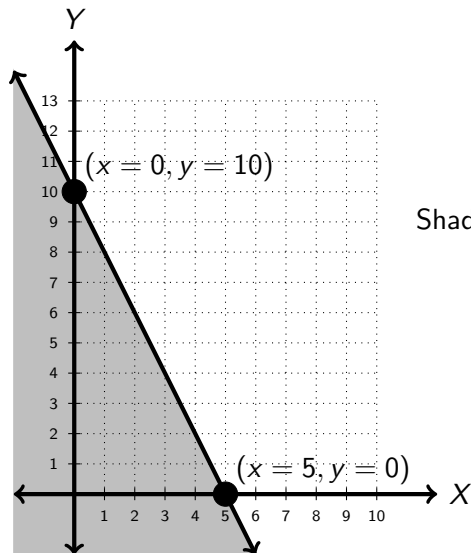
$$y = 0, \ 200x = 1000, \ x = 5$$

3.1: Graphing inequalities

$$200x + 100y = 1000$$

Connect the dots

3.1: Graphing inequalities

$$200x + 100y \leq 1000$$

Shade the region

3.1: Graphing inequalities

- First graph the "equality", that is, graph the line

$\Rightarrow$ Find two points on the line and then draw the connection

- Next graph the inequality, that is, shade the region

$\Rightarrow$ Choose a point not on the lines and see if it is on the correct side

- For example (0,0) is on the correct side since

$$(200)(0) + (100)(0) \leq 1000$$

3.1: Is it realistic?

- Our region is very large.

- Some points don't make sense for a single purchaser:

⇒ (2.5, 3.5) means buy 2.5 Xylophones and 3.5 Yukuleles ($850)

- But maybe it makes sense as an average or a strategy

- Some points don't make any sense for any purchaser:

⇒ (−10, −20) means buy -10 Xylophones . . . (-$4000)
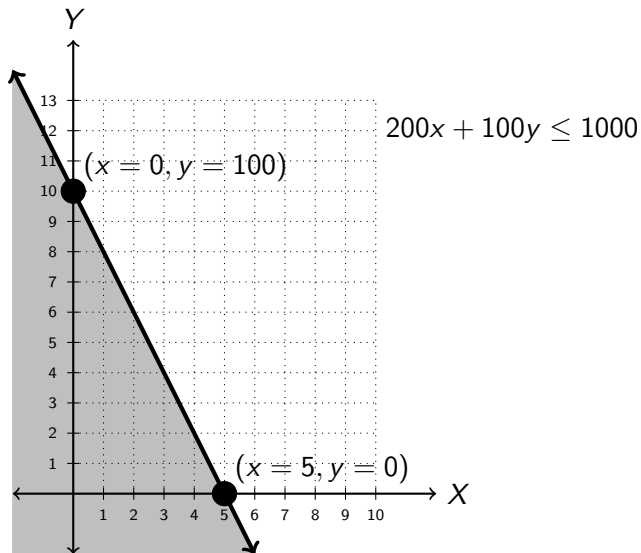
3.1: Systems of inequalities

- We also need some sanity: $X \geq 0$ and $Y \geq 0$

- So we have a system of inequalities:

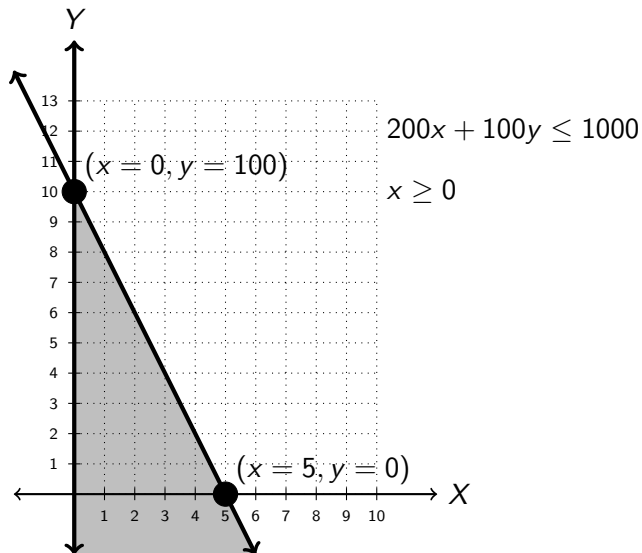$$\begin{cases} 200X + 100Y \leq 1000 \\ X \geq 0, Y \geq 0 \end{cases}$$

- Not enough for just one to be true!

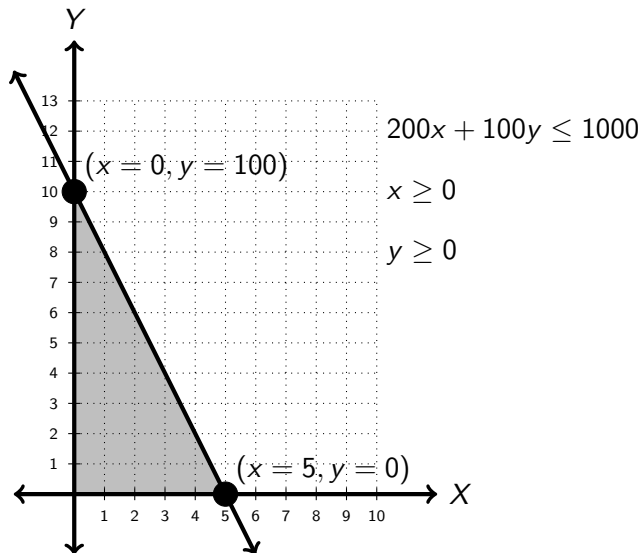$\Rightarrow$ $(500, 0)$ would be very expensive (\$100,000) and noisy!

3.1: Graphing systems of inequalities
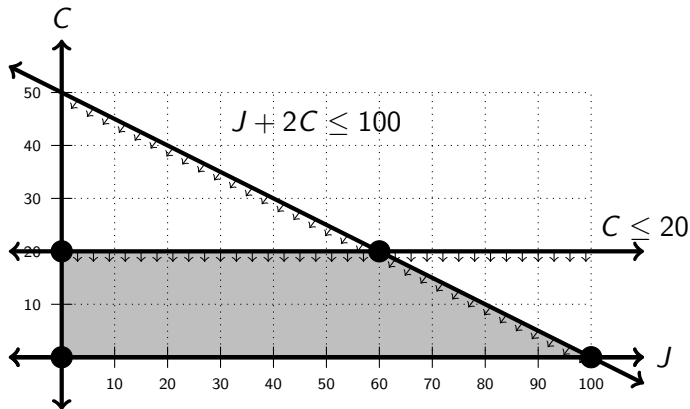
3.1: Graphing systems of inequalities

$200x + 100y \leq 1000$

$x \geq 0$

$(x = 0, y = 100)$

$(x = 5, y = 0)$

3.1: Graphing systems of inequalities

$200x + 100y \leq 1000$

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$y \geq 0$

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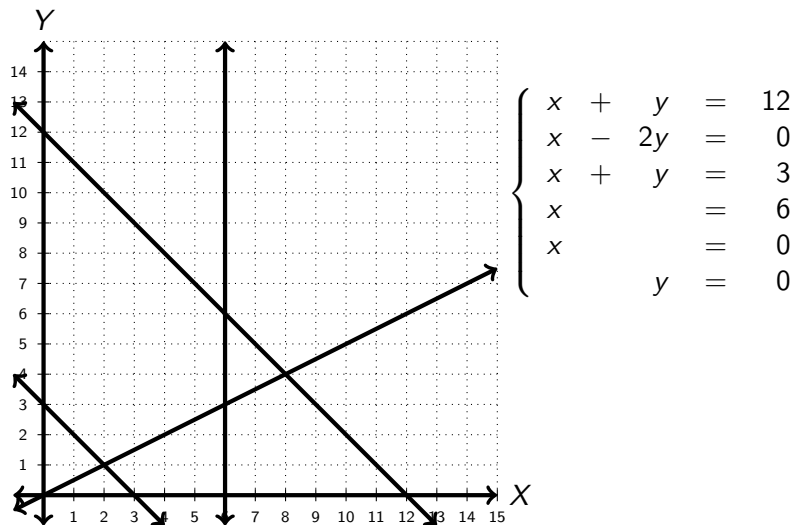
3.1: Graphing systems of inequalities

- Graph each equality (line)

- Figure out which side of the line is good

- Shade the region that is on the correct side of **all** lines

- Alternatively: figure out which of the pieces is good

3.1: Graphing systems of inequalities

Draw little arrows to show which side is good.
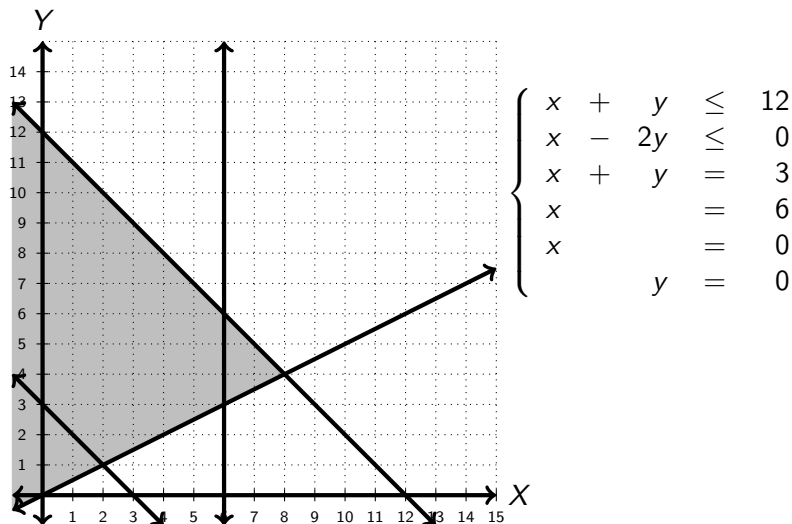
3.1: Graphing systems of inequalities

$$\begin{cases} x & + & y & = & 12 \\ x & - & 2y & = & 0 \\ x & + & y & = & 3 \\ x & & & = & 6 \\ x & & & = & 0 \\ & & y & = & 0 \end{cases}$$

Draw all the lines, then check each inequality.

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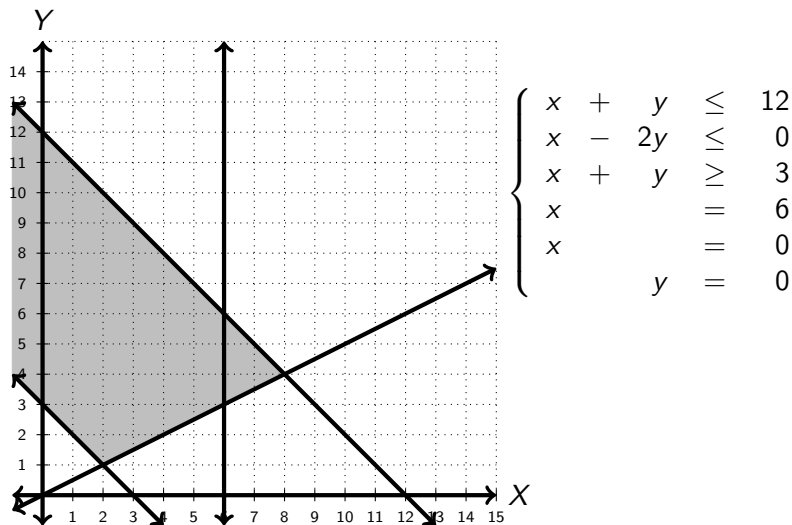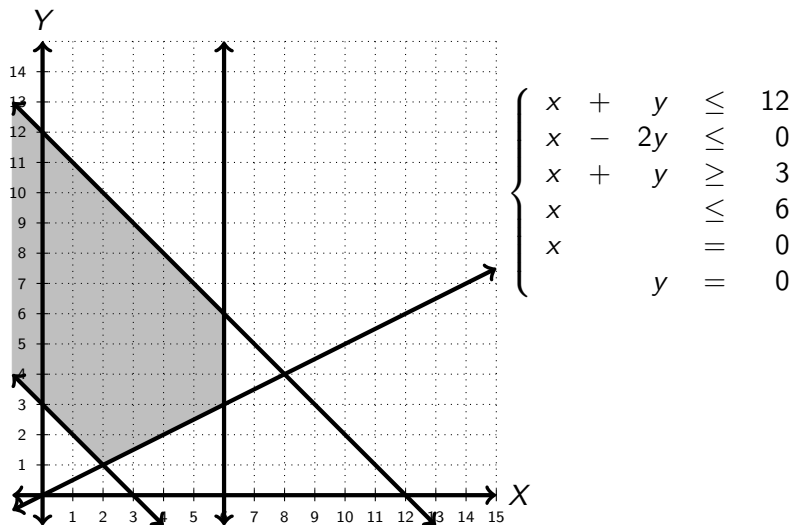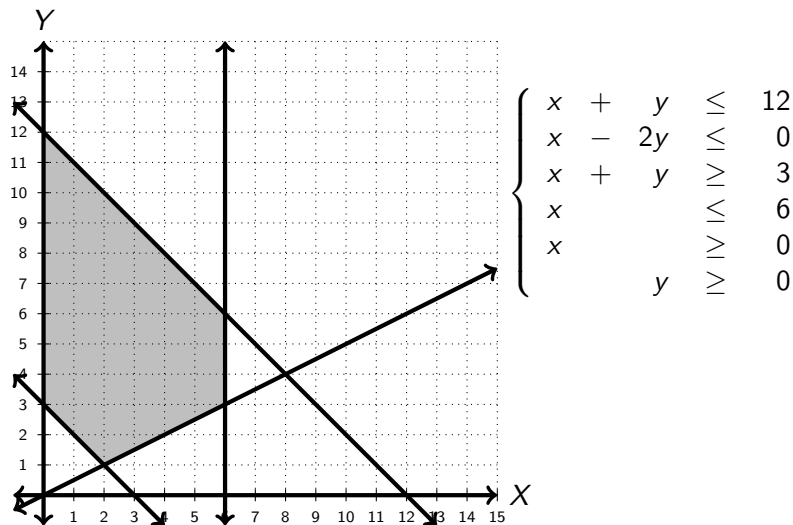
Draw all the lines, then check each inequality. Too many regions!

3.1: Graphing systems of inequalities
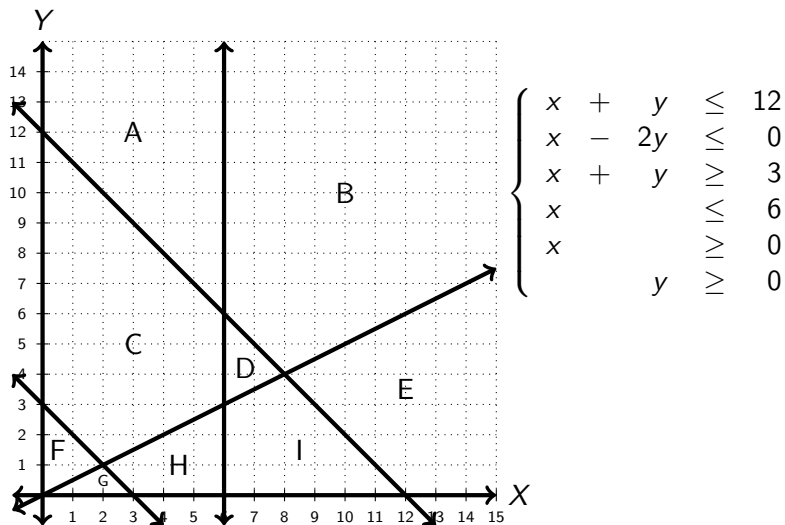
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Check a point in each region to find the right one.

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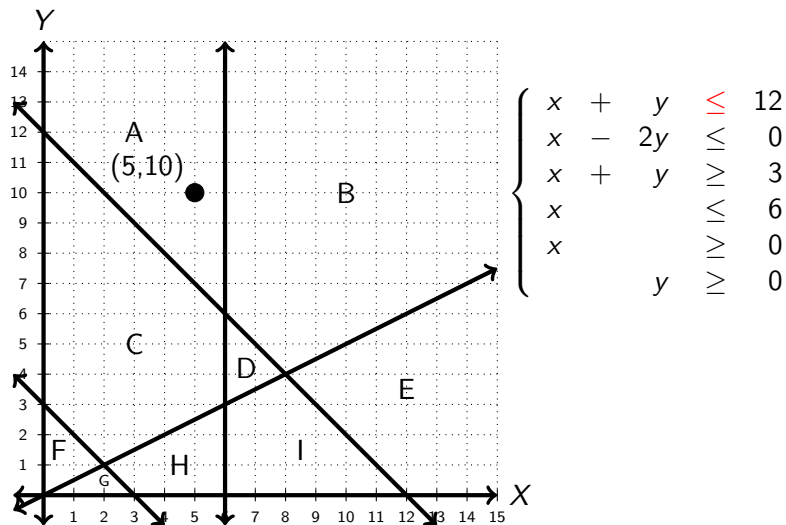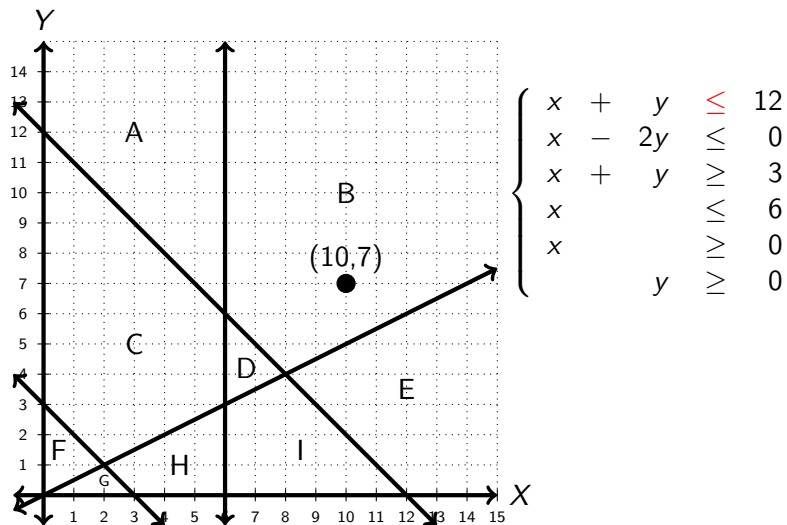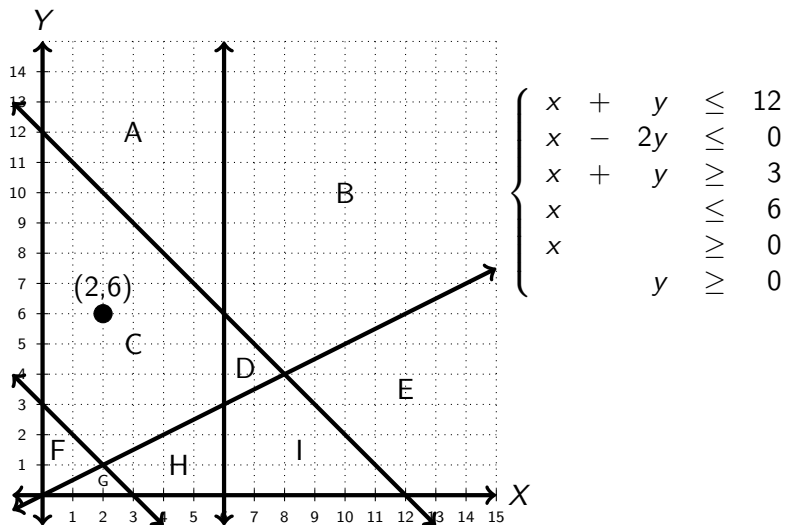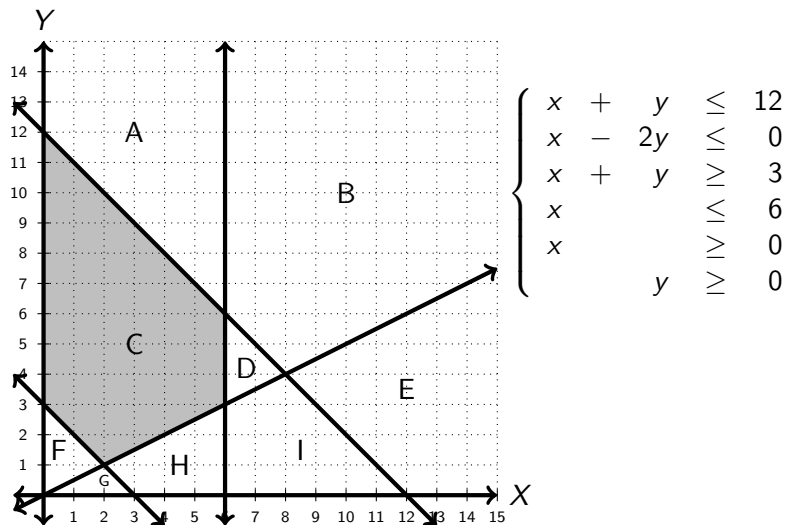
Check a point in each region to find the right one. Yay!

3.1: Finding corners

$$\begin{cases} x & + & y & \leq & 12 \\ x & - & 2y & \leq & 0 \\ x & + & y & \geq & 3 \\ x & & & \leq & 6 \\ x & & & \geq & 0 \\ & & y & \geq & 0 \end{cases}$$
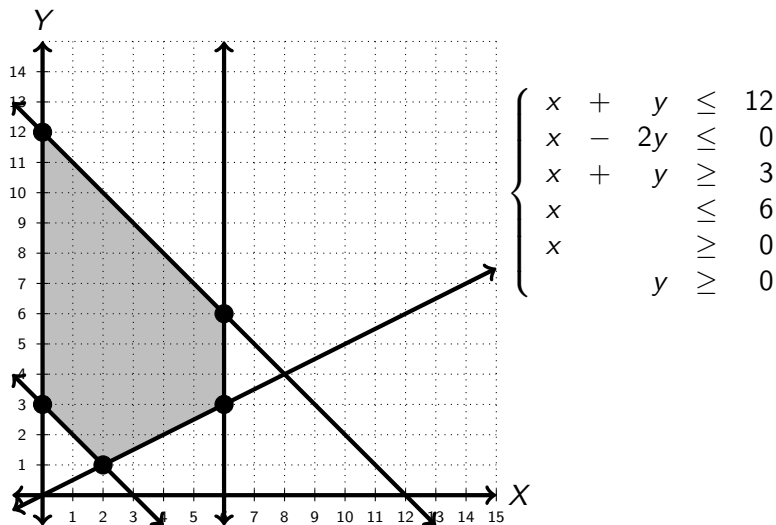
Intersect each pair of lines, and check it satisfies other inequalities

3.1: Finding corners

- For each pair of lines, find the intersection

- Then check that intersection satisfies the rest of the inequalities

- Not all intersections are corners!

- All corners are intersections.

- Intersections are just 2×3 RREF problems!

- How many angles does a triangle have?

3.1: How many corners are there?

- How many angles does a triangle have?

- How many sides does a triangle have?

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- How many angles does a quadrangle have? A quadrilateral?

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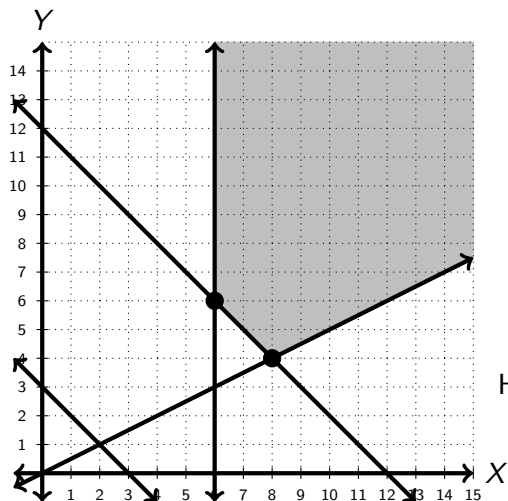- An *n*-sided polygon has *n* angles too!

3.1: Where's the missing corner?
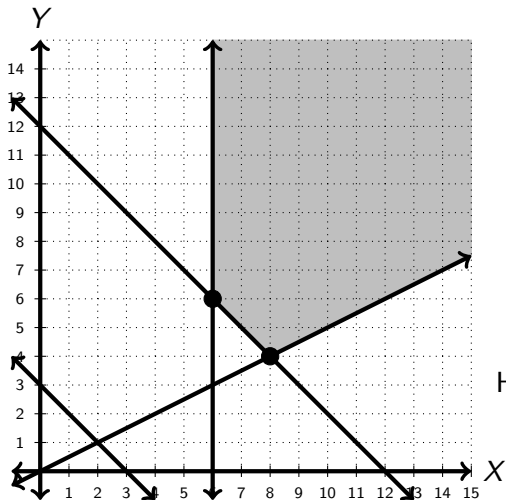
How many edges?

How many corners?

3.1: Where's the missing corner?

How many edges?

How many corners?

- This is called **unbounded** and it means we need to handle the "missing corner" specially.