DEPARTMENT OF MATHEMATICS

Ma 162 Second Exam March 5, 2012

Instructions: No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If your answer is not in the box or if you have no work to support your answer, you will receive no credit. The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	**Maximum Score**	**Actual Score**
1	11	
2	11	
3	11	
4	11	
5	12	
6	11	
7	11	
8	11	
9	11	
Total	100	

NAME: _____ **Section:** _____

Last four digits of Student ID: _____

(Practice version; updated 2012-03-01 19:30)

Show your work. Answers with no work receive no credit.

1. Use the matrices given to answer the following arithmetic problems. If a problem cannot be solved, explain why it cannot be solved.

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \quad D = \begin{bmatrix} 10 & 11 \\ 12 & 13 \\ 14 & 15 \end{bmatrix}$$

(a) $A + B$

(b) $B + C$

(c) $C + D$

(d) $7A + 2B$

(e) AB

(f) BC

(g) AD

Show your work. Answers with no work receive no credit.

2. Use the matrices given to answer the following arithmetic problems. If a problem cannot be solved, explain why it cannot be solved.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 4 & 8 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} \qquad C^{-1} = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 0 \end{bmatrix}$$

(a) A^{-1}

(b) If $CX = B$, then find X explicitly.

3. Zach's goal is to get the most bird-songs out of his limited resources. He has four limited resources: 396 sunflower seeds, 482 millet grains, 246 kernels of corn, and 593 crumbs of bread. He has designed two products for the birds: the Cheep dinner and the Squawky dinner. He predicts 2 lovely bird-songs for every Cheep dinner, and 3 loud bird-songs for every Squawky dinner. The resource usage and song production of each his products is listed in the following table:

	Seeds	Grains	Kernels	Crumbs	Songs
each Cheep dinner	4	6	2	8	2
each Squawky dinner	8	4	6	1	3
Available	396	482	246	593	

Zach's friend suggests that Zach feed the birds 51 Cheep dinners and 24 Squawky dinners. **Describe the effects of such a business decision by filling in the form:**

Number of Cheep dinners: 51
Number of Squawky dinners: 24

Number of songs: _____

Leftover seeds: _____

Leftover grains: _____

Leftover kernels: _____

Leftover crumbs: _____

Give your own recommendation in the next form to increase the number of songs using only his limited resources #6 and #7 may be useful.

Number of Cheep dinners: _____

Number of Squawky dinners: _____

Number of songs: _____

Leftover seeds: _____

Leftover grains: _____

Leftover kernels: _____

Leftover crumbs: _____

4. Soup Parlour has asked you to maximize profit by setting production goals for their soup mixing factory without using more than their limited supplies of chicken stock, beef stock, and vegetable stock. Their stores sell Meaty soup, Leafy soup, and Soupy soup (see table for ingredients). The three soups earn the Soup Parlour profits of \$1.20, \$1.30, and \$1.50 per bowl, but there is limited demand for the soups: 1200, 600, and 900 bowls.

	ounces of Chicken stock	ounces of Beef stock	ounces of Vegetable stock	bowls of Demand	Profit
each bowl of Meaty	1	6	1	1200	\$1.20
each bowl of Leafy	0	0	8	600	\$1.30
each bowl of Soupy	3	2	2	900	\$1.50
Available	3400	6800	5014		

Set up this problem below. You will do one step of its solution on #8, and read and interpret the final answer on #9.

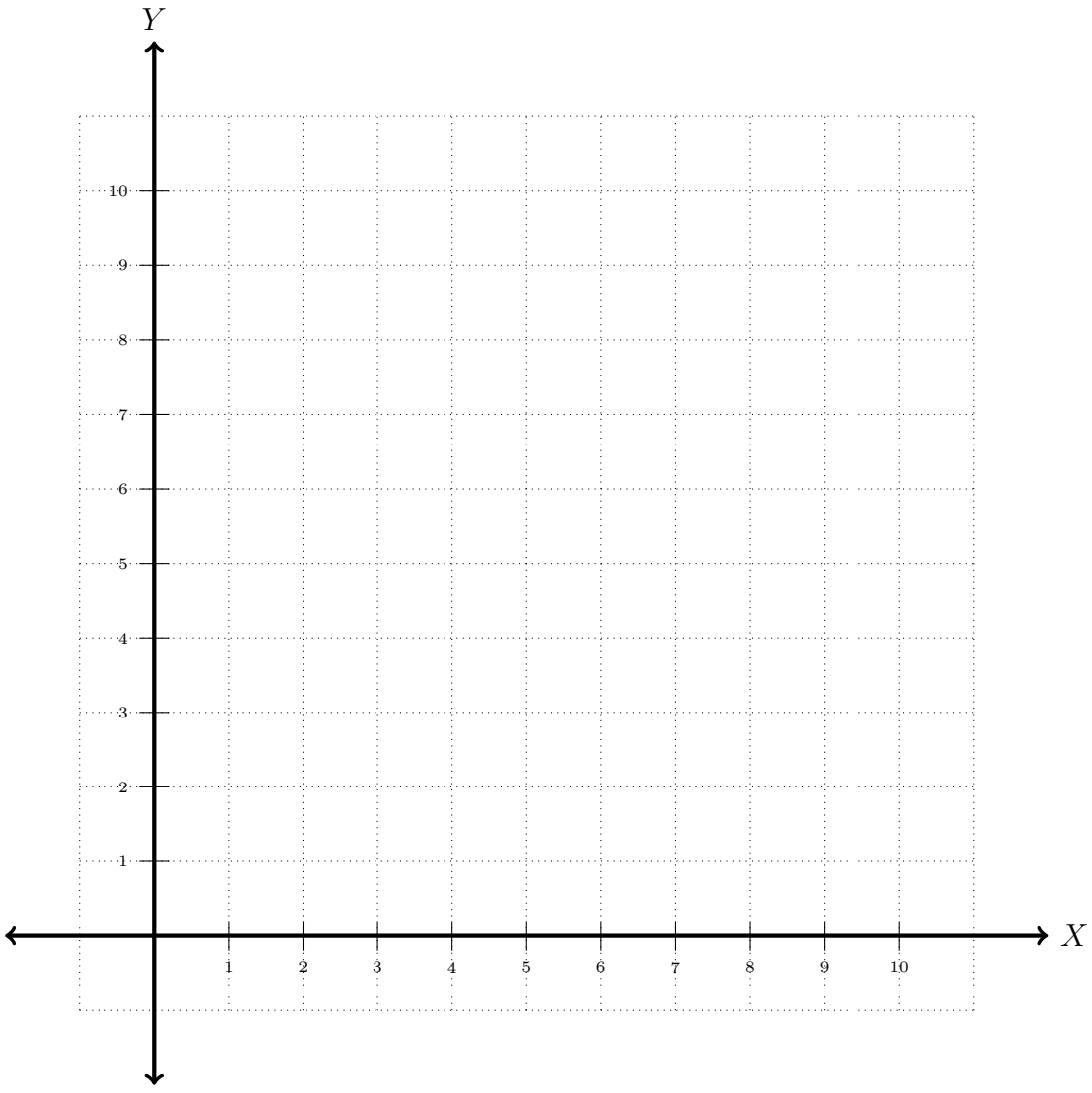
Variables:

Constraints:

Objective:

5. Graph the feasible region for the following LPP. You will be graded on three aspects: correctly drawn edges, correctly shaded region, and correctly labelled corners. (The numbers in this problem are not related to either word problem, but you may find the picture on #6 to be a good model of clear edges, corners, and labels).

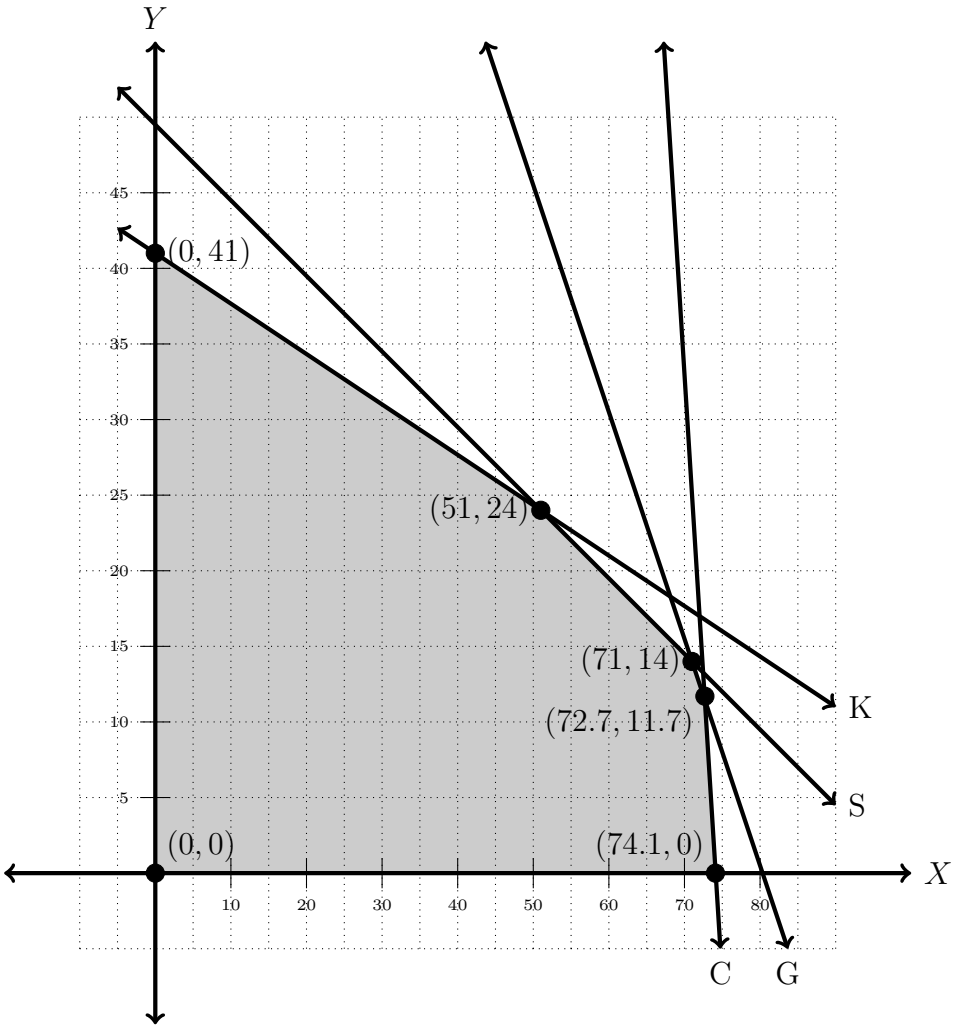
Maximize $S = 2x + 3y$ subject to $\left\{ \begin{array}{l} 2x \leq 8 + 4y \\ 4x + 4y \leq 40 \\ 5x + 10y \leq 80 \\ 6y \leq 9x + 30 \end{array} \right\}$ and $x \geq 0$, $y \geq 0$.



Is this region bounded or unbounded?

6. List the corners, determine if the region is bounded or unbounded, and find the maximum value of P.

Maximize $P = 2x + 3y$ subject to $\begin{cases} S: & 4x + 8y \leq 396 \\ G: & 6x + 4y \leq 482 \\ K: & 2x + 6y \leq 246 \\ C: & 8x + y \leq 593 \end{cases}$ and $x \geq 0$, $y \geq 0$.



X	Y	P

Is this region bounded or unbounded?

The maximum value of P is _____ and it occurs at ($x =$ _____, $y =$ _____).

7. Write down the (standard, primal) simplex tableau corresponding to the problem:

$$\text{Maximize } P = 2x + 3y \text{ subject to } \begin{cases} S\text{:} & 4x + 8y \leq 396 \\ G\text{:} & 6x + 4y \leq 482 \\ K\text{:} & 2x + 6y \leq 246 \\ C\text{:} & 8x + \ \ y \leq 593 \end{cases} \text{ and } x \geq 0,\ y \geq 0.$$

8. Apply one full step of the simplex algorithm. Circle your pivot, write out your row operations, and write down the next tableau. Explain why that next tableau is or is not final. (This is the table for #4 resulting from the three decisions: (1) Leafy is good, but limited by the demand for Leafy, (2) Soupy is good, but limited by the remaining supply of vegetable stock, (3) hungry Leafy customers are actually ok, as this decreases hungry Soupy customers)

	M_{eaty}	L_{eafy}	S_{oupy}	C_{hicken}	B_{eef}	$V_{eg.}$	H_{ungry} M	H_{ungry} L	H_{ungry} S	Profit	RHS
C	1	0	0	1	0	0	0	0	-3	0	700
B	6	0	0	0	1	0	0	0	-2	0	5000
P	0	0	1	0	0	0	0	0	1	0	900
HM	1	0	0	0	0	0	1	0	0	0	1200
L	1/8	1	0	0	0	1/8	0	0	-1/4	0	1607/4
HL	-1/8	0	0	0	0	-1/8	0	1	1/4	0	793/4
Profit	-415/4	0	0	0	0	65/4	0	0	235/2	1	374455/2

Is this a final tableau? Why or why not?

9. Read the answer from the following finished tableau (based on #4). Give the location of the maximum, the maximum itself, and the resulting surpluses.

Use the word problem in #4 to give a plain English version of the answer for your supervisor at the Soup Parlour. Be sure to include the recommended decision, its important effect (the "bottom line"), and some information on the slack variables.

	M	L	S	C	B	V	HM	HL	HS	Profit	RHS
	1	0	0	-1/8	3/16	0	0	0	0	0	850
	0	0	0	-3/8	1/16	0	0	0	1	0	50
	0	0	1	3/8	-1/16	0	0	0	0	0	850
	0	0	0	1/8	-3/16	0	1	0	0	0	350
	0	1	0	-5/64	-1/128	1/8	0	0	0	0	308
	0	0	0	5/64	1/128	-1/8	0	1	0	0	292
	0	0	0	995/32	775/64	65/4	0	0	0	1	269540

M = _____ C = _____ HM = _____

L = _____ B = _____ HL = _____

S = _____ V = _____ HS = _____

Profit = _____ (Be careful about units for the Profit)

Plain English recommendation:

Higher level evaluation: Does the Soup Parlour need more supplies or more marketing right now?