MA162: Finite mathematics

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April 16, 2012

Schedule:

- HW 7A, 7B due Fri, April 20, 2012
- HW 7C due Fri, April 27, 2012
- Final exam, Wed May 2, 2012 from 8:30pm to 10:30pm

Today we will cover 7.2: Probability

Final Exam

- Chapter 7: Probability
  - Counting based probability
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  - Empirical probability
  - Conditional probability
- Cumulative
  - Ch 2: Setting up and reading the answer from a linear system
  - Ch 3: Graphically solving a 2 variable LPP
  - Ch 4: Setting up a multi-var LPP
  - Ch 4: Reading and interpreting answer form a multi-var LPP

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- The event "rolling saved us money" is all those pairs that total to more than 6.
- There are 21 such pairs, and if all pairs are equally likely (the dice are fair), then that is $\frac{21}{36} = \frac{7}{12} \approx 58\%$

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- 16 ways to win, 32 ways total, so $\frac{16}{32} = \frac{1}{2} = 50\%$ chance
- Explicitly:
  HHHHH, HHHHT, HHHTH, HHHTT, HHTTT, HTHHH, HTTTH, HTTTT, THHHH, THHHT, THTTT, TTHHH, TTTHH, TTTHT, TTTTH, TTTTT

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- Hence it should be about $\frac{1}{8} + \frac{3}{8} = 50\%$ of the time to get either 1 or 3 heads
- It should be the same for getting an odd number of tails, right? Tails, heads, what is the difference?
- But you either get an odd number of heads, or an odd number of tails, and not both, so each should be about equally likely: 50%

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- If each bulb is independent, that is $(0.1\%)^{700} \approx 0\%$ chance of this happening

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- Total is: $0.844 = 84.4\%$ chance that at most one breaks, so not too bad. Every 6 weeks you'll have a light out and no replacement, but not too bad.

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- What are the odds that 10 is enough?
- The odds of none going out is $(99.9\%)^{7000} \approx 0.1\%$,
  exactly one are $7000 \cdot (0.1\%)(99.9\%)^{6999} \approx 0.6\%$,
  exactly two are $\frac{7000 \cdot 6999}{2} \cdot (0.1\%)^2(99.9\%)^{6998} \approx 2.2\%$,
  ...

0	1	2	3	4	5	6	7	8	9	10
0.1	0.6	2.2	5.2	9.1	12.7	14.9	14.9	13.0	10.1	7.0

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- Total is: $0.902 = 90.2\%$ chance that at most ten break, so really we're even more certain to be ok now; every 10 weeks we'll be short a bulb.

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- The larger the population, the less extreme the whims of fortune
- This is why insurance is important; the risk to one person is great
- The risk to 10,000 people is quite small, much less than 10,000 times the risk of one

Round table

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- Sample space is:
  ABCD, ABCE, ABDC, ABDE, ABEC, ABED, ACBD, ACBE, ACDB, ACDE, ACEB, ACED, ADBC, ADBE, ADCB, ADCE, ADEB, ADEC, AEBC, AEBD, AECB, AECD, AEDB, AEDC, BCDE, BCED, BDCE, BDEC, BECD, BEDC

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- 12 bad out of 30 total is 40% chance for showers (of fists)