MA111: Contemporary mathematics

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August 27, 2012

Entrance Slip (due 5 min past the hour):

- Convert the following linear ballots into a preference schedule:

	Ballot
1st	A
2nd	B
3rd	C

	Ballot
1st	A
2nd	C
3rd	B

	Ballot
1st	B
2nd	C
3rd	A

	Ballot
1st	C
2nd	B
3rd	A

- Who has the most first place votes?
- Who has the most last place votes? Is that good?

Schedule:

- Online HW 1A,1B is due Friday, Aug 31st, 2012.
- Online HW 1C,1D,1E,1G is due Friday, Sep 7th, 2012.
- Exam 1 is Monday, Sep 17th, during class.

Today we discuss strategic voting.

Expectations

- I expect you to have turned in your entrance slip now

- I expect you to have read and understood pages 4-5 (Ch 1.1)

- I expect you to have read pages 2-9.

- I expect you to have completed online HW 0 and most of 1A

- I expect you to have been to office hours (Friday 4pm to 5pm) if you had questions

- I expect you to be at my office hours today (2pm to 4pm; mathskeller) if you still have questions

Context: We have the preferences, how do we decide?

- Ok, let's look at a simple election:

	35	33	32
1st	A	B	C
2nd	B	C	B
3rd	C	A	A

- If everyone just votes for their favorite, who wins?

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- "Is that fair?"

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- Can those 32 C-B-A people do something to "fix" the election?

Activity: Can you do it?

- Now that we realize not everybody tells the truth in politics, can we still win?

- Divide into groups of 8-10 (first three rows, then split down the middle and aisles) and decide how your group is going to vote to get the best outcome

- Here are your rankings:

Front

A	C	C	D
C	A	B	B
D	D	D	C
B	B	A	A

A	B	B	D
B	C	C	B
C	A	D	A
D	D	A	C

- Pretend that if your first place winner wins, you get full points; if your second place winner wins, you get 90%; then 80%; then 70%

Fast: Four kinds of things in this chapter

- This chapter is about:

  - Vote counting methods (plurality)

  - Fairness criteria (majority, condorcet)

  - Voting strategies (abandon the loser?)

  - Neat examples (like the three we did today)

- Do not mix up the kinds!

- We will use words in a technical sense.
  Precision allows certainty.

Fast: Plurality

- The simplest vote counting method is **plurality**

- It is a way to decide who wins: **most first place votes**

Definition
A candidate is a **plurality winner** if it receives the most first place votes.

Definition
The **plurality vote counting method** declares a plurality winner the winner if there is one, and otherwise declares a tie. See exercises #11, #12 on page 31 for some sample tie-breaking rules.

- It is an answer to Friday's quiz, but not the only answer

- It is kind of "fair" and kind of "unfair"

Fast: Why is it unfair?

- In the entrance slip, A was the plurality winner, 4 out of 10 first place votes, but the other two candidates only had 3 first place votes.

Ballot	
1st	A
2nd	B
3rd	C

Ballot	
1st	A
2nd	C
3rd	B

Ballot	
1st	B
2nd	C
3rd	A

Ballot	
1st	C
2nd	B
3rd	A

- But A is hated! 6 out of 10 rank A last!

- Plurality is unfair!

- We need to be **precise**.

Fast: A fairness criterion

- **Majority loser criterion:** (exercise #75 page 40)

Definition

A candidate is said to be a **majority loser** if strictly more than 50% of the voters rank it last.

Definition

A vote counting method is said to **satisfy the majority loser criterion** if a majority loser never wins.

Theorem

The plurality vote counting method does not satisfy the majority loser criterion.

Proof.

In the entrance slip example, the majority loser is A, but A wins according to plurality. It happened once, so "it never happens" is not true. □

Fast: Plurality is fair!

- People who are well-liked will win plurality, so that is fair.

Definition

A **majority winner** is a candidate who has strictly more than 50% of the first place votes.

Definition

A vote counting method is said to **satisfy the majority fairness criterion** if a majority winner never loses according to this method.

Theorem

The plurality vote counting method satisfies the majority fairness criterion.

- Can you explain why?

Fast: Condorcet fairness criterion

- Political instability results when a candidate is elected who cannot beat one of the other candidates head-to-head

	35	33	32
1st	A	B	C
2nd	B	C	B
3rd	C	A	A

- A vs B: 35 to 65
  A vs C: 35 to 65
  B vs C: 68 to 32

- B always wins! Even if A is elected, B could challenge the leadership.

- Most people (65%) would agree B is better than A.

Fast: Condorcet criterion

- We try to be precise:

Definition

A candidate is a **Condorcet winner** if it beats every other candidate head-to-head.

Definition

A vote counting method is said to **satisfy the Condorcet fairness criterion** if a Condorcet winner is never declared a loser.

Theorem

The plurality vote counting method does not satisfy the Condorcet fairness criterion.

Proof.

In the first strategic example, B is a Condorcet winner, but B loses according to the plurality vote counting method. It happens once, so not "never". □

Assignments

- Read and understand pages 4-9 (Ch 1.1 - 1.2).

- Read pages 2-11.

- Be able to do exercises #1-8, page 29-30 (do odds, and check work) and start on #62 page 38 (ignore part (iii))

- Online HW for 1B due Friday.

- **Exit slip:**

  Explain the difference between a **plurality winner** and a **majority winner**.

- First two done (and neighbors agree), write an explanation on the board.