# MA111: Contemporary mathematics

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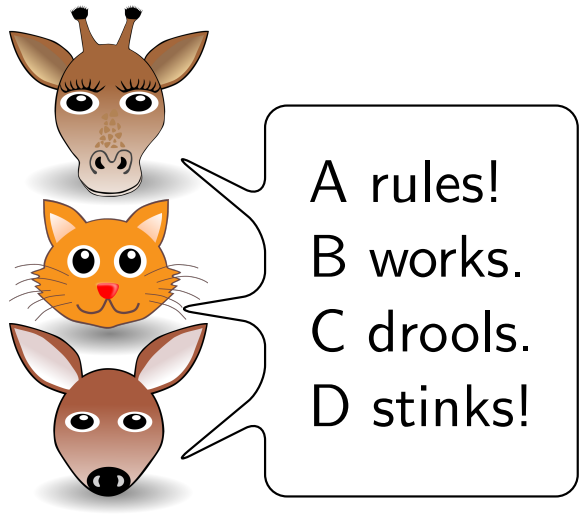
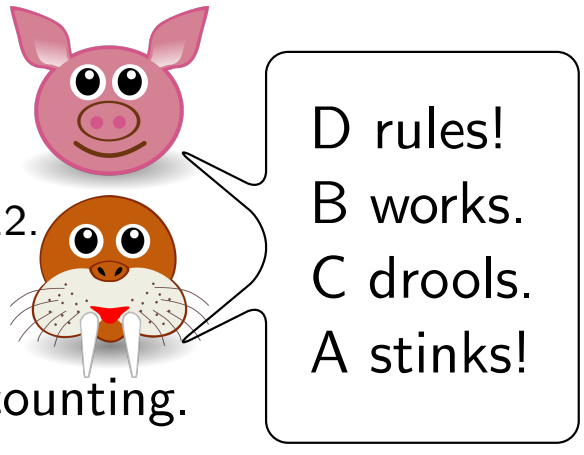



Entrance Slip (due 5 min past the hour):

- Which option (A, B, C, or D) do we choose to stop the fighting?
- Does the majority agree?




Schedule:

- Online HW 1A,1B is due Friday, Aug 31st, 2012.
- Online HW 1C,1D,1E,1G is due Friday, Sep 7th, 2012.
- Exam 1 is Monday, Sep 17th, during class.

Today we discuss a way to get more out of vote counting.

# Expectations

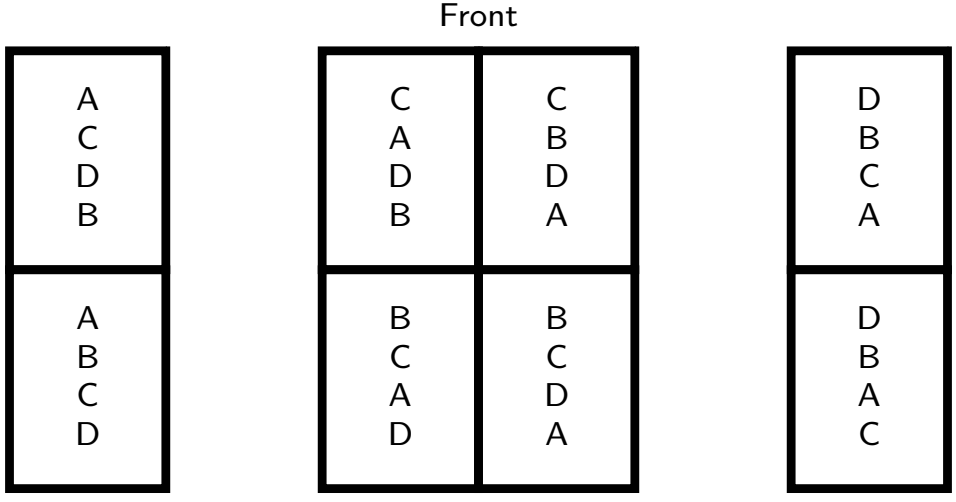
- I expect you to have turned in your entrance slip now
- I expect you to have read and understood pages 4-9 (Ch 1.1 - 1.2)
- I expect you to have read pages 2-11.
- I expect you to have completed HW 1A and most HW 1B
- I expect you to have been to office hours (Monday 2pm to 4pm; mathskeller) if you had questions

# Context

- The plurality vote counting method was both "fair" and "unfair":
  - It satisfied the majority (winner) fairness criterion
  - It failed the majority loser fairness criterion
  - It failed the Condorcet fairness criterion
- It failed because it ignored second place votes.
- Many groups succeeded on Monday by paying attention to second place votes.
- Maybe we can make a more reliable system by giving points to second place?

# Activity: Gaming the system again

- Each group will turn in a piece of paper with their top TWO votes (ranked, 1st then 2nd)
- Each 1st place vote will get 2 points, and each 2nd place vote will get 1 point.
- Here is your "payoff schedule" (top wins=100%, 2nd=90%, etc.)




- Decide how your group will vote. Each group will secretly vote (1st and 2nd place) in 5 or 10 minutes and we will total them up.

# Fast: Borda count mechanics

- The full Borda count gives points to third place, fourth place, etc.

| | **Ballot** |
|---|---|
| 1st | A |
| 2nd | B |
| 3rd | C |
| 4th | D |

- Each ballot gives each candidates some points:
  one point for every candidate equal to or lower than them
- A is on top, all 4 candidates are equal or lower, so 4 points
- B is second, 3 candidates are equal or lower, so 3 points
- C is third, so 2 points
- D is fourth, so 1 points
- Total up the points from all the ballots to get the final score
- Borda count winner is the one with the high score

# Fast: Borda count example

- Here is a small example:

| | 10 | 5 | 2 |
|---|---|---|---|
| 1st | A | B | B |
| 2nd | B | C | D |
| 3rd | C | D | C |
| 4th | D | A | A |

- The first column ballots give A 4 points each,
  10 ballots like that, so 40 points

- A gets $(10)(4) + (5)(1) + (2)(1) = 40 + 5 + 2 = 47$ points total
  B gets $(10)(3) + (5)(4) + (2)(4) = 30 + 20 + 8 = 58$ points total
  C gets $(10)(2) + (5)(3) + (2)(2) = 20 + 15 + 4 = 39$ points total
  D gets $(10)(1) + (5)(2) + (2)(3) = 10 + 10 + 6 = 26$ points total

- B is the clear winner!

# Fast: Borda count is unfair

- A is majority winner!

| | 10 | 5 | 2 |
|---|---|---|---|
| 1st | A | B | B |
| 2nd | B | C | D |
| 3rd | C | D | C |
| 4th | D | A | A |

B is Borda count winner

**Theorem**

*Borda count does not satisfy the majority fairness criterion.*

- A is also a Condorcet winner. (Why?)

**Theorem**

*Borda count does not satisfy the Condorcet fairness criterion.*

- Borda count elects "moderate" candidates,
  even if an "extreme" candidate has a majority!

# Fast: Borda count is fair

- Borda count will not elect a "bad" candidate!

**Theorem**

*Borda count satisfies the majority loser fairness criterion.*

- In fact, it satisfies a stricter criterion:

**Definition**

A candidate is said to be a **Condorcet loser** if it loses every head-to-head competition.

**Definition**

A vote counting method is said to **satisfy the Condorcet loser fairness criterion** if a Condorcet loser never wins.

**Theorem**

*Borda count satisfies the Condorcet loser fairness criterion.*

# Fast: Condorcet loser photo

- After the last match-up:

# Assignments

- Read pages 12-16 and reread pages 10-11.
- Book exercises #21-#26 are all good. You can now try full #62.
- **Exit Slip:** Who is the:
  1. Plurality winner
  2. Our activity winner
     (2 pts for 1st, 1 pt for 2nd)
  3. Condorcet loser

| | 4 | 3 | 2 |
|---|---|---|---|
| 1st | A | D | C |
| 2nd | B | C | B |
| 3rd | C | B | D |
| 4th | D | A | A |

- Who **should** win? (Or at least who is the Borda count winner?)
- When you are done discussing your exit slip,
  pass it to the end of the row.
- Raise your hand for homework problems.