MA111: Contemporary mathematics

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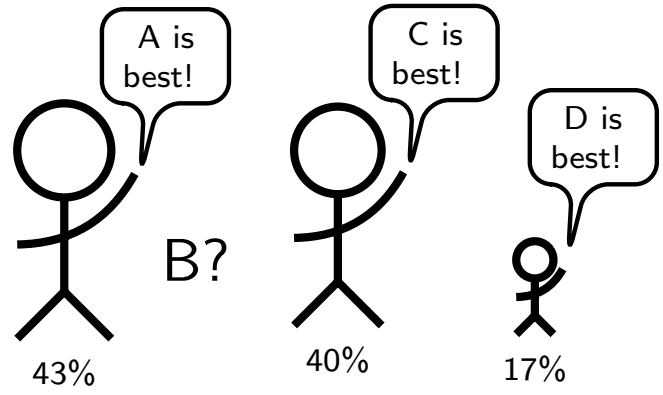


Entrance Slip (due 5 min past the hour):
A is going to win, so C and D just give up.
B keeps a stiff upper lip (alone).
Who wins with only A and B left?

	10	7	5	5	3
1st	A	C	C	D	A
2nd	B	B	B	B	B
3rd	C	A	D	A	D
4th	D	D	A	C	C

Schedule:

- Online HW 1A,1B is due Friday, Aug 31st, 2012.
- Online HW 1C,1D,1E,1G is due Friday, Sep 7th, 2012.
- Exam 1 is Monday, Sep 17th, during class.

Today we discuss what happens when bad candidates go away

Expectations

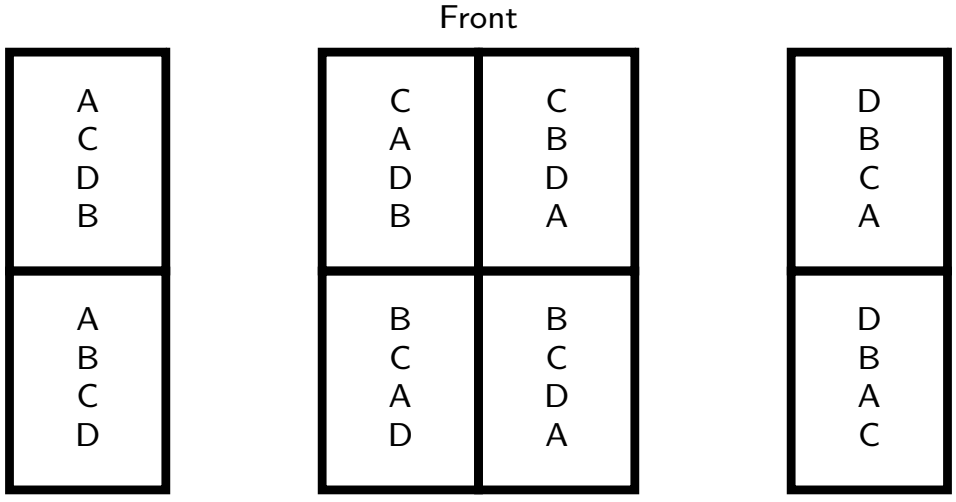
- I expect you to have turned in your entrance slip now
- I expect you to have read and understood pages 4-11 (Ch 1.1 - 1.3)
- I expect you to have read pages 2-16.
- I expect you to have completed HW 1A and HW 1B and to have started on HW 1C
- I expect you to be at my office hours today (4pm to 5pm in Mathskeller) if you have questions
- We will do "Condorcet" stuff all day on Monday.

Context

- Borda count elected a moderate candidate and never elects a really bad candidate
- Why don't we just get rid of the bad candidates to begin with?
- If they aren't going to win, why don't they just give up
- We want to see why this leads to complete insanity.

Activity: Most extreme elimination activity

- Here is your "payoff schedule" (top wins=100%, 2nd=90%, etc.)



- Game I: Vote who to eliminate. Most ("last place") votes is eliminated. Repeat until only one left.

- Game II: As a group decide on a ranking. Then using those rankings we'll eliminate the candidates with the most last place votes, then amongst the survivors, we'll eliminate the most hated, etc. until only one left.

Fast: Plurality with elimination mechanics

- Elimination methods all work about the same:
  1. Use a vote counting method to rank all the candidates
  2. Only keep the top so-and-so many
  3. Repeat until only one candidate left
- Plurality with elimination:
  1. Rank candidates using number of first place votes
  2. Keep all but the single worst candidate
  3. Repeat until only one candidate left
- These methods only require the voters to vote once!
- The linear ballot allows us to remove candidates without needing to ask the voters to vote again.

Fast: Plurality with elimination example

- Example:

	10	7	5	5	3
1st	A	C	C	D	A
2nd	B	B	B	B	B
3rd	C	A	D	A	D
4th	D	D	A	C	C

- First ranking: A with 13, C with 12, D with 5, B with 0.
  B is eliminated.

	10	7	5	5	3
1st	A	C	C	D	A
2nd	C	A	D	A	D
3rd	D	D	A	C	C

- Second ranking: A with 13, C with 12, D with 5
  D is eliminated.

	10	7	5	5	3
1st	A	C	C	A	A
2nd	C	A	A	C	C

- Final ranking: A with 18, C with 12
  C is eliminated. A wins.

Fast: Plurality with elimination is fair

- Plurality with elimination satisfies one of the fairness criteria:

Theorem

Plurality with elimination satisfies the majority (winner) fairness criterion.

- That is about it as far as fairness.
- However, if there are no ties for last place during the eliminations:
  - A majority loser loses.
  - A condorcet loser loses.

Fast: Plurality with elimination is unfair

- In this election, a Condorcet winner (B) loses while the winner is a Majority and Condorcet Loser!

	2	1	1	1
1st	A	B	C	D
2nd	B	C	B	B
3rd	C	D	D	C
4th	D	A	A	A

However, this required an exact tie!

- Even without ties a Condorcet winner can lose:
B is eliminated, then C is eliminated, then A wins.

	2	2	1
1st	A	C	B
2nd	B	B	A
3rd	C	A	C

But A vs B: 2 to 3
B vs C: 3 to 2.
B is a Condorcet winner!

Fast: Plurality with elimination is INSANE

- The biggest problem with all point-based elimination vote counting methods (plurality = 1 point for first place; Wednesday we did two other point systems) is they are insane. Stark raving mad.
- Should the first column tell the truth?

	2	6	5	4
1st	A	A	B	C
2nd	C	B	A	B
3rd	B	C	C	A

- Please work out who wins Plurality with Elimination if everyone tells the truth. First one to confirm with your neighbor, please put it on the board.
- How does the first column feel about that?
- Now how can the first column lie to do better?

Fast: Monotonicity "fairness" criterion

- The precise version:

Definition

A vote counting method is said to **satisfy the monotonicity criterion** if a winner remains a winner even when a voter ranks the winner more favorably.

Theorem

Plurality with elimination does not satisfy the monotonicity criterion.

Theorem

"Survivor" style elimination does not satisfy the monotonicty criterion.

Theorem

(Smith, 1973) No points based elimination method satisfies the monotonicity criterion, but every points based vote counting method does.

Assignments

- Read pages 16-20 and reread pages 12-16.
- Book exercises #29, 31, 73
- **Exit Slip:** Eliminate candidate A from the following preference schedule:

	7	6	5	4	3
1st	A	B	C	D	A
2nd	B	A	B	A	B
3rd	C	D	D	C	D
4th	D	C	A	B	C

- When you are done discussing your exit slip, pass it to the end of the row.
- Raise your hand for homework problems.