MA111: Contemporary mathematics

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Entrance Slip (due 5 min past the hour):

- Can a Condorcet winner get no first place votes?
  (Give an example to show it can, or explain why it cannot.)
- Can a Condorcet winner have the most last place votes?
  (Give an example to show it can, or explain why it cannot.)

Schedule:

- Online HW 1C,1D,1E,1G is due Friday, Sep 7th, 2012.
- Exam 1 is Monday, Sep 17th, during class.

Today we look at Condorcet nearly-winners.

Review of the vote counting methods

- We have discusssed 3 major (and 2 more minor) vote counting methods:
  - (1.2) **Plurality:** most first place votes wins
  - (1.3) **Borda count:** highest average ranking wins
  - **2nd place is half credit:** like plurality, but 2nd place counts as half a 1st place
  - (1.4) **Plurality with elimination:** eliminate the candidate with the least first place votes
  - **Survivor:** eliminate the candidate with the most last place votes
- Each method had good features and bad features.
- To be precise, we defined "fairness criteria"
  a vote counting method either satisfied them or not

Review of the fairness criteria

- We have discussed 3 major (and 2 more minor) fairness criterion:
  - **Majority (winner) fairness criterion:** If a candidate has more than 50% of the first place votes, he should win.
  - **Majority loser fairness criterion:** If a candidate has more than 50% of the last place votes, he should lose.
  - **Condorcet (winner) fairness criterion:** If a candidate can beat every other candidate head-to-head, he should win.
  - **Condorcet loser fairness criterion:** If a candidate is beaten by every other candidate head-to-head, he should lose.
  - **Monotonicity:** If a candidate wins one election, then he should also win an election where the only difference is a voter ranked the winner higher. ("more first place votes should help")

Review: How do they do?

- Here is a table describing how well our vote counting methods do:

	MW	ML	CW	CL	Mo	IIA
PI	Y	N	N	N	Y	N
BC	N	Y	N	Y	Y	N
$2 = \frac{1}{2}$	N	N	N	N	Y	N
PE	Y	*	N	*	N	N
Su	N	Y	N	*	N	N
PC	Y	Y	Y	Y	Y	N

- Today we will cover the gray row and column
- The * means mathematically no, but practically yes

Activity: Finding Condorcet winners

- Examine the preference schedule:

	7	7	3	3
1st	E	B	B	E
2nd	B	C	G	B
3rd	G	G	E	D
4th	C	D	F	G
5th	F	A	C	C
6th	A	E	D	A
7th	D	F	A	F

- In your group, split up the work to check all the head-to-head matchups
- Who is closest to being a Condorcet winner?
- How can you organize the winners to find the best one?

Fast: Pairwise comparison mechanics

- Look at **every** head-to-head competition
- Winners of head-to-heads get 1 point, ties get 1/2 point
- Most points wins
- One head-to-head:
  A vs B: 6+3+1 vs 5+3+2, tie!
  A vs C: 6+3+1 vs 5+3+2, tie!
  B vs C? Do they tie too?

	6	5	3	3	2	1
1st	A	B	B	C	C	A
2nd	B	C	A	A	B	C
3rd	C	A	C	B	A	B

Fast: Pairwise comparison mechanics

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- Most points wins
- One head-to-head:
  A vs B: 6+3+1 vs 5+3+2, tie!
  A vs C: 6+3+1 vs 5+3+2, tie!
  B vs C: 6+5+3 vs 3+2+1, B wins

	6	5	3	3	2	1
1st	A	B	B	C	C	A
2nd	B	C	A	A	B	C
3rd	C	A	C	B	A	B

- Total scores:

	A	B	C
Wins	0	1	0
Ties	2	1	0
Total	1	1.5	0

So B is the Pairwise Comparison winner

Fast: Pairwise comparison is very fair

- Pairwise comparison satisfies all of our old criteria:

Theorem

Pairwise comparison satisfies:

- *the majority (winner) fairness criterion,*
- *the majority loser fairness criterion,*
- *the Condorcet (winner) fairness criterion,*
- *the Condorcet loser fairness criterion,*
- *the monotonicity criterion*

- However, it has two main problems: ties and disqualification

Fast: Interlude and a silly story

- WAITRESS: Will you have the Apple or the Blueberry pie
- SIDNEY: The Apple please.
- WAITRESS: Oh, we also have Cherry pie.
- SIDNEY: In that case, I'll have the Blueberry.
- We know pie is irrational, but is Sidney?

Fast: Independence of Irrelevant Alternatives

- Sidney ranks pie (Apple, Blueberry, Cherry) using 7 criteria:

	Texture	Aroma	Gooeyness	Nutrition	Crumbliness	Flavor	Beauty
1st	A	A	C	C	B	B	B
2nd	C	C	A	A	A	A	A
3rd	B	B	B	B	C	C	C

- The best flavor is the one highest ranked (amongst those available) in the most categories
- Apple versus Blueberry: Apple wins on the first four categories!
- Apple versus Blueberry versus Cherry: B wins on the last three!
- Rational, but weird.

Fast: Independence of Irrelevant Alternatives

- We prefer our voting methods to be less weird:

Definition

A vote counting method is said to **satisfy the independence of irrelevant alternatives criterion** if a winner remains a winner even if a losing candidate is disqualified.

Theorem

Plurality does not satisfy the IIA criterion.

- In fact, none of our methods satisfy the IIA.

Fast: IIA nearly always fails

- In a 3-candidate race where not everyone wins,
  IIA means we can eliminate a loser to get a 2-candidate race
- In a 2-candidate race, there is only one sane way to decide!
- But consider Condorcet's Paradox:

	40%	35%	25%
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

- If A is not a winner, then IIA+majority says B wins (75%)
  If B is not a winner, then IIA+majority says C wins (60%)
  If C is not a winner, then IIA+majority says A wins (65%)
- Problem: If B wins, then both A and C are not winners, so C wins, but wait. . .
- Solution: Everyone wins! YAY!

Assignment

- Reread and understand pages 2-20
- Read pages 27-28
- Good book homeworks #1, 3, 17, 23, 33, 59, 60, 61, 62, 68, 72, 73, 74, 75, 79
- Exit slip: Give a single example where each of the following statements is the view of a (sizable) majority:
  - A is better than B
  - B is better than C
  - C is better than D
  - D is better than E
  - E is better than A

  Which candidate is best?