MA111: Contemporary mathematics

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Entrance Slip (due 5 min past the hour):

- Can a candidate be both a majority winner and a majority loser?
  (Give an example to show it can, or explain why it cannot.)
- Can a candidate be both a Condorcet winner and a Condorcet loser?
  (Give an example to show it can, or explain why it cannot.)

Schedule:

- Practice exam 1 is due in class on Friday Sep 14.
- Exam 1 is Monday, Sep 17th, during class.

Today we review our fairness criteria.
Wednesday we'll cover Arrow's theorem.

Context: Fairness means never be unfair

- In this class we have defined our fairness criteria as:

  never do this bad thing

- To show a vote counting method is (ever) unfair:
  Find a **single** time when it does the wrong thing

- To show a vote counting method is (always) fair:
  Give a convincing argument that it can never do the wrong thing

- Unfair is easier to check your own work: just check the example

- Fair is harder, and it is better to convince your study group
  Critique each other's arguments

Activity: Check these two vote counting methods

- The worksheet has two vote counting methods:
  - "First or last": 1st place votes gain one point, last place votes lose one point
  - "Final four": Do A vs B and C vs D as semi-finals, then the winners face-off in a final match
- You must check 4 of the 6 fairness criteria for each method:
  - Majority winner: A majority winner **should** win, but does it always?
  - Majority loser: A majority loser **should** lose, but does it always?
  - Condorcet winner/loser are similar
  - Monotonicity: If you raise the ranking of a winner, but leave the rest alone, the winner **should** still win, but does it always?
  - IIA: If you leave the ranking of the winner alone, but change the rest, the winner **should** still win, but does it always?

Activity: Answers for "first-or-last"

- Majority winner: FAIL

	3	2
1st	A	B
2nd	B	C
3rd	C	D
4th	D	A

- Majority loser: TRUE! (my argument is mathy)
- Condorcet winner: FAIL (same as majority winner)
- Condorcet loser: FAIL

	2	2	1	1
1st	A	B	C	D
2nd	B	D	D	A
3rd	C	C	A	C
4th	D	A	B	B

- Monotonicity: TRUE!

bump winner from 2nd to 1st: good for winner, bad for the other guy, everyone else stays the same
bump winner from last up: good for winner, bad for the new last, everyone else stays the same
bump winner from middle to middle: changes nothing

- IIA: FAIL

	1	1	1
1st	A	B	C
2nd	B	C	A
3rd	C	D	D
4th	D	A	B

A:0, B:0, **C:1**, D:−1 ⟶

	1	1	1
1st	B	B	C
2nd	A	C	A
3rd	C	D	B
4th	D	A	D

A:−1, **B:2**, C:1, D:−2

Alternatively, the two losers A and D could drop out, then B would win instead.

Activity: Answers for "final four"

- Majority/Condorcet winner: TRUE!

  he wins round 1, he wins round 2, he wins!

- Majority/Condorcet loser: TRUE!

  he loses round 1, he loses round 2, he loses!

- Monotonicity: TRUE!

  The bump only helps him in his own round 1, and it does not affect the other round 1, so round 2 is the same or even better for the winner

- IIA: FAIL

	1	1	1
1st	A	D	C
2nd	B	C	A
3rd	D	A	B
4th	C	B	D

AvB: A, CvD: D, AvD: **A** ⟶ AvB: A, CvD: C, AvC: **C**

	1	1	1
1st	A	C	C
2nd	B	D	A
3rd	D	A	B
4th	C	B	D

Alternatively just disqualify that loser D.

Review of the vote counting methods

- We have discusssed 4 major (and 2 more minor) vote counting methods:
  - (1.2) **Plurality:** most first place votes wins
  - (1.3) **Borda count:** highest average ranking wins
  - **2nd place is half credit:** like plurality, but 2nd place counts as half a 1st place
  - (1.4) **Plurality with elimination:** eliminate the candidate with the least first place votes
  - **Survivor:** eliminate the candidate with the most last place votes
  - (1.5) **Pairwise comparison:** Most head-to-head victories wins! Ties count half-credit.
- Each method had good features and bad features.
- To be precise, we defined "fairness criteria" a vote counting method either satisfied them or not

Review of the fairness criteria

- We have discussed 4 major (and 2 more minor) fairness criterion:
  - (1.2) **Majority (winner) fairness criterion:** If a candidate has more than 50% of the first place votes, he should win.
  - **Majority loser fairness criterion:** If a candidate has more than 50% of the last place votes, he should lose.
  - (1.2) **Condorcet (winner) fairness criterion:** If a candidate can beat every other candidate head-to-head, he should win.
  - **Condorcet loser fairness criterion:** If a candidate is beaten by every other candidate head-to-head, he should lose.
  - (1.4) **Monotonicity:** If a candidate wins one election, then he should also win an election where the only difference is a voter ranked the winner higher. ("more first place votes should help")
  - (1.5) **IIA:** Disqualifying a loser should not affect who wins.

Review: How do they do?

- Here is a table describing how well our vote counting methods do:

	MW	ML	CW	CL	Mo	IIA
PI	Y	N	N	N	Y	N
BC	N	Y	N	Y	Y	N
$2 = \frac{1}{2}$	N	N	N	N	Y	N
PE	Y	*	N	*	N	N
Su	N	Y	N	*	N	N
PC	Y	Y	Y	Y	Y	N

- Notice how none of the methods satisfy all the criteria.
- Kenneth Arrow showed you cannot have a Y in both MW and IIA for any method.
- The * means mathematically no, but practically yes

Assignments and exit slip

- Reread and understand 2-20, 27-28.
- Read the slides for Wed before Wednesday's class

  (they are the same as last Friday's slides that we skipped)
- Good book homeworks #1, 3, 17, 23, 33, 59, 60, 61, 62, 68, 72, 73, 74, 75, 79
- Download and start on the practice exam (after it is announced)
- **Exit slip:**

  Does the plurality vote counting method satisfy the majority loser fairness criterion?

  Give an argument why it (always) does, or give an example where it fails (at least this once).

  Be sure to explain your answer clearly.

Entrance slip answers

- Can a candidate be both a majority winner and a majority loser?

  No! If you have more than half of the first place votes, your other place votes share less than half of the voters, so last place has strictly less than half of the voters.

- Can a candidate be both a Condorcet winner and a Condorcet loser?

  No! A candidate that wins every head-to-head match does not lose any head-to-head matches, so it cannot lose them all.

- Technically: if there is only one candidate then first place = last place, so he will always be both the majority winner and majority loser with 100% of both. Similarly, since there aren't any head-to-head matchups, I guess he wins and loses all of them.

Exit slip answers

- Does the pluralty vote counting method satisfy the majority loser fairness criterion?

  No! Z gets 60% of last place votes, but still wins!

	1	1	1	2
1st	A	B	C	Z
2nd	B	C	A	A
3rd	C	A	B	B
4th	Z	Z	Z	C