MA111: Contemporary mathematics

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Entrance Slip (due 5 min past the hour):

- If a candidate has 80% of the first place votes: will he (A)lways, (S)ometimes, or (N)ever win under Plurality?
- What about under Borda count?

Schedule:

- Practice exam due Friday in class.
- Exam 1 is Monday, Sep 17th, during class.

Today we cover Arrow's theorem and practice finding interesting schedules.

Context: Conclusion and Arrow's theorem

Theorem (Arrow, 1950)

A vote counting method for more than 2 candidates cannot satisfy both the majority fairness criterion and the IIA fairness criterion.

- A major consequence is that you cannot ignore the losers in an election (unless you also ignore the majority of voters).

Theorem (Gibbard-Satterhwaite, 1973)

A vote counting method for more than 2 candidates in which "honesty is the best policy" for voters is either a dictatorship (one voter decides) or custom (one candidate always wins).

- A major consequence is that you cannot ignore the benefits of lying (unless the voting system ignores the votes).
- By lying about how they feel about the losers, voters may be able to get a better result.

Activity: Case studies in fairness (violations)

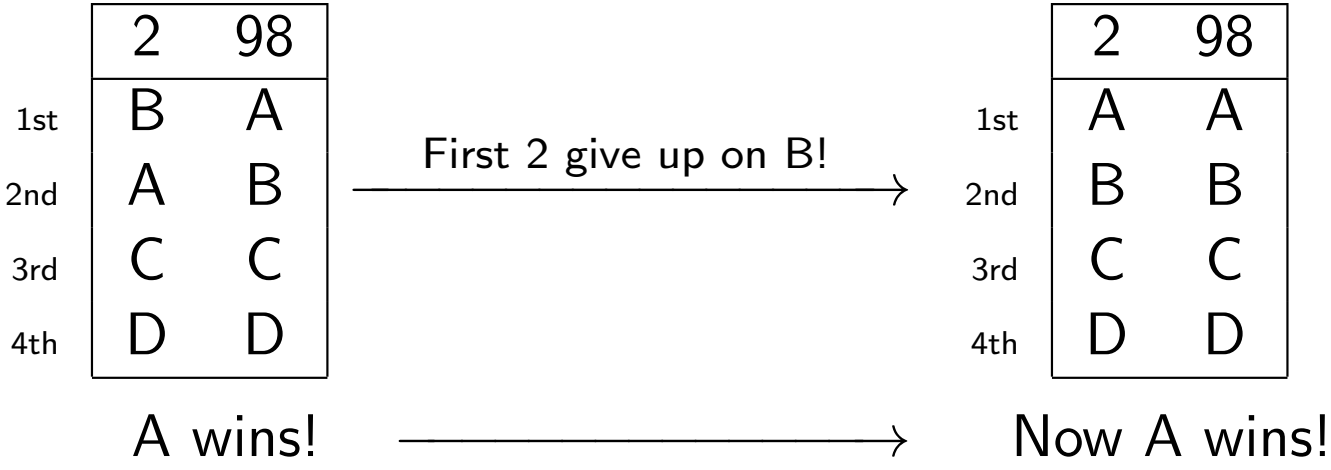
- The worksheet has 10 situations on it (similar to 1G homework).

  Decide which criterion is violated (if any)

  Decide which voting methods could have been used for such an unfair election.

- You may find it helpful for the exam to write down a sample preference schedule for each one.

  Number 10:



Assignment and exit slip

- Work the practice exam before next class.
  Be ready to present problems at the board.

- **Exit slip:** How well do plurality and Borda count agree in this election:

	3	2	2	2
1st	B	C	A	A
2nd	C	D	D	D
3rd	D	B	B	C
4th	A	A	C	B

- Compare plurality with elimination versus pairwise comparison.

- Does it even make sense to ask which candidate is best for the group?

Entrance slip answers

- If a candidate has 80% of the first place votes: will he (A)lways, (S)ometimes, or (N)ever win under Plurality?

  Always. Only 20% of the first place votes can go to any other candidate, so the candidate with 80% definitely has the most.

- What about under Borda count?

  Sometimes. If the other 20% don't hate the guy, he should do fine.

	80	20
1st	A	C
2nd	B	A
3rd	C	B

	80	20
1st	A	B
2nd	B	F
3rd	C	C
4th	D	D
5th	E	E
6th	F	A

A:260, B:220, C:120 A:500, B:520, C:400, D:300, E:200, F:180

So the 80% can win as on the left, or lose as on the right.

Exit slip answers

	3	2	2	2
1st	B	C	A	A
2nd	C	D	D	D
3rd	D	B	B	C
4th	A	A	C	B

- How well do plurality and Borda count agree?

  Plurality ranks the candidates $A > B > C > D$
  (A:4, B:3, C:2, D:0)
  Borda ranks the candidates $D > C > B > A$
  (A:21, B:22, C:23, D:24)
  Exactly opposite!

- Compare plurality with elimination versus pairwise comparison.

  Plurality with elimination ranks them $B > A > C > D$
  Pairwise comparison ranks them $B = C = D > A$
  A is the Condorcet loser but last to be eliminated!

- Does it even make sense to ask which candidate is best for the group?

  It is hard for me to see who is best. A is the worst, but A is the best.