

MA111: Contemporary mathematics

Jack Schmidt
University of Kentucky

September 19, 2012

Entrance Slip (due 5 min past the hour):

- Which is the better deal: (a) \$10 off or (b) 5% off?
- Which is the better deal: (a) regular price or (b) 10% markup first, then 10% discount

SCHEDULE:

- HW 10.1 is due Friday, Sep 21st, 2012.
- HW 10.2,10.3 is due Friday, Sep 28th, 2012.
- HW 10.6 is due Friday, Oct 5th, 2012.
- The second exam is Monday, Oct 8th, during class.
- First exam grades on Friday.

Today we will look at percentages.

Context: How do numbers change?

- The first entrance slip question answer is “it depends!”
- If the item only costs \$12, then \$10 off is a great deal!
- If the item costs \$1000, then 10% off is \$100 off, much better!
- If you buy two of the \$12 items, do you get \$10 off each?
- We need to be able to convert between percentages and dollar amounts.

Activity: Which is the better deal?

All prices are for the same product.

- (A) \$100 original price. 10% off sale. 6% tax.
- (B) \$ 90 original price. No sale. 7% tax.
- (C) \$110 original price. 20% off sale. 6% tax.
- (D) \$100 original price. \$10 off coupon. 6% tax.
- (E) \$150 original price. 30% off sale. 6% tax.

Fast: Percentages as decimals

- A **percentage** is a way of writing a fraction:

17% means “17 out of 100”

- Decimals are nice and easy:

$$17\% = 0.17,$$

$$1\% = 0.01,$$

$$100\% = 1.00,$$

$$200\% = 2.00$$

- To take a percentage of something, you multiply:

17% of \$100 is \$17 out of \$100

Fast: Percentage increase, intro

- Adding percentages is a little weird:

\$100 + 17% is kind of abbreviated: 17% of what?

- Better to write $\$100 + (17\% \text{ of } \$100)$ or even

$$\$100 + (0.17)(\$100) = \$100 + \$17 = \$117$$

- What percentage of \$100 is \$100?
- If we have \$100 of \$100 and get another 17% of \$100,
what percentage of \$100 do we have?
- Even better to write $\$100(1.17)$, 117% of the \$100.

Fast: percentage increase

- A **percentage increase** is a specific way of writing down a relationship between numbers: 17% increase means the new number is 117% of the old number
- We just multiply by $1 + 0.17 = 1.17$
- So a 17% increase in \$100 is $(\$100)(1.17) = \117
- A 17% increase in \$200 is $(\$200)(1.17) = \234
- Notice how if we double the original, we also double the increase

$\$100 \rightarrow \200 means $\$17 \rightarrow \34

Fast: percentage decrease

- A **percentage decrease** is another relationship:

17% decrease means the new number is $100\% - 17\% = 83\%$ of the old

- So a 17% decrease in \$100 is $(\$100)(0.83) = \83

- A 17% decrease in \$200 is $(\$200)(0.83) = \166

- Notice how doubling works in a nice predictable manner

$\$100 \rightarrow \200 means $-\$17 \rightarrow -\34 means $\$83 \rightarrow \166

Fast: mixed increases and decreases (entrance slip, part 2)

- Which is better: (a) regular price or (b) 10% markup first, then 10% sale
- One reasonable answer is that it should be the same. 10% up, 10% down
- Well if it starts at \$100, it goes up to \$110. How far does it go down?

Fast: mixed increases and decreases (entrance slip, part 2)

- Which is better: (a) regular price or (b) 10% markup first, then 10% sale
- One reasonable answer is that it should be the same. 10% up, 10% down
- Well if it starts at \$100, it goes up to \$110. How far does it go down?
- What if we did the discount first, then the markup. Is it \$100 (regular), \$99 (like up-down), or \$101 (the opposite)?
- What does this have to do with $1 - x^2$?

Fast: mixed increases and decreases

- Using multiplication makes these problems easy:
- 7% up, 7% down, 5% up, 2% down:
 $(1.07)(0.93)(1.05)(0.98) = 1.02396$
Whole thing is just a 2.396% increase
- So which is better (a) 10% up first, then 10% down, or (b) 10% down first, then 10% up?
- In both cases we have $(1 + 0.10)(1 - 0.10) = (1 - 0.10)(1 + 0.10)$
- This is just $(1 + x)(1 - x) = (1 - x)(1 + x) = 1 - x^2$.

Up-down of the same amount is a bit smaller than regular price

Assignments and exit slip

- Read pages 366-370, examples 10.7 and 10.8

(Example 10.9: we do credit cards later)

- Reread and understand pages 362-366 (10.1)

- Online homework 10.1 due Friday; same as book examples and today

- **Exit slip:**

(A) What is 10% of \$100?

(B) An \$8 item is first marked up 25%, then marked down 25%. What is the final price?