MA111: Contemporary mathematics

Jack Schmidt
University of Kentucky

September 26, 2012

Entrance Slip (Show Your Work; due 5 min past the hour):

- Write out how to calculate \$100 increased by 2% twelve times

Schedule:

- HW 10.2,10.3 is due Friday, Sep 28th, 2012.
- HW 10.6 is due Friday, Oct 5th, 2012.
- The second exam is Monday, Oct 8th, during class.

Today we will cover 10.3, compound interest.

Context: interest on interest

- Suppose you need money a year later, not now
- A bank will pay you simple interest, 2% per month
- If you put in \$100, each month you get \$2 in interest

  By the end, you've got \$100+\$24 = \$124
- Can you do better?
- Spend the \$2 at the bank across the street

  They'll give you \$0.04 per month for 11 months

  By the end, you've got \$124.44
- Why would the first bank want to lose your business?

Activity: compounding interest

- If banks only offered 2% per month simple interest paid monthly

  What is the most you could make in 12 months? (just using interest)

- Would you be better off using a bank that offered 25% per year simple interest, if it paid out at the end of a year?

- How would you compare per month and per year interest rates in general?

Activity: Proposed answer

- Each month you re-invest, and the total grows by 2%
- **Entrance slip answer:** $\$100(1.02)^{12} = \$126.82$

  \$24 interest on the original,

  Another \$2.82 total from the interest on interest
- More than 10% of the interest was from interest on interest
- How do we compare?

Just run the money through both investments.

- Monthly we got 26.82% interest in 12 months; better than 25%
- This is why we learn to calculate: don't waste a year waiting for the bank statements to tell you which is better; calculate now and then enjoy the benefits

Fast: 10.3: Compound interest formulas

- The following formula is important enough to memorize:

P = Present value
F = Future value
p = periodic compound interest rate
T = number of periods

$$F = P(1 + p)^T$$

- Same as repeatedly doing simple interest for 1 period

Fast: 10.3: Monthly example

- Our activity example:

  $P = \$100$

  $F = ?$

  $p = 0.02$ (per month)

  $T = 12$ (months)

  $F = P(1 + p)^T = \$100(1 + 0.02)^{12} = \$100(1.02)^{12}$

- $F = \$126.82$

Fast: 10.3: Yearly example

- Compare to the other bank in the activity:

  $P = \$100$

  $F = ?$

  $p = 0.25$ (per year)

  $T = 1$ (year)

  $F = P(1 + p)^T = \$100(1 + 0.25)^1 = \$100(1.25)$

- $F = \$125.00$, not as big

Fast: 10.3: More compound interest formulas

- These formulas are not worth memorizing, in my opinion

P =	Present value
F =	Future value
APR = r =	annual, nominal, compound interest Rate
n =	Number of periods per year
t =	number of years
APY = r_{eff} =	annual effective Yield (what you actually get)

$$F = P\left(1 + \frac{r}{n}\right)^{(nt)}$$

$$APY = \left(1 + \frac{r}{n}\right)^{(n)} - 1$$

- If $n = \infty$, then we get:

$$F = Pe^{(rt)}$$

Fast: APR versus APY Example

- A bank won't usually call it 2% per month

  Often they call it 24% APR, (2% per month times 12 months per year)

  But we saw that our \$100 became \$126.82, more than 24%

- What percent per year was it?

  $P = \$100$

  $F = \$126.82$

  $p = ?$ (per year)

  $T = 1$ (year)

- $\$126.82 = \$100(1 + p)$

  $\$126.82 = \$100 + \$100p$

  $\$26.82 = \$100p$

- $p = \$26.82/\$100 = 0.2682 = 26.82\%$ APY

Assignment and exit slip

- Reread and understand 10.3
- Read 10.6 (you may want to very lightly skim 10.4 and 10.5)
- **Exit slip:** Which is the better deal if you need the money 24 months from now?

  (a) 2% per month for 24 months

  (b) 26.82% per year for 2 years (so you get the interest after 12 months, and re-invest)
- What does this tell you about using the APY?