

MA111: Contemporary mathematics

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Entrance Slip (Show Your Work; due 5 min past the hour):

- How much can you borrow today at 2% per month interest, if you are willing to pay back \$100 next month?
- How much can you borrow today at 2% per month interest, if you are willing to pay back \$100 next month AND the month after?

SCHEDULE:

- HW 10.6 is due Friday, Oct 5th, 2012.
- The second exam is Monday, Oct 8th, during class.

Today we cover longer installment loans.

Context: Time travel for money

- The first entrance slip question is 10.3: compound interest:

$$P = ?$$

$$F = \$100$$

$$p = 0.02 \text{ per month}$$

$$T = 1 \text{ month}$$



- $P = F / (1 + p)^T = \$100 / 1.02 = \98.04 now, for \$100 in one month
- To move money from the future to the present, divide by the 1.02
- How about the second entrance slip question? A little trickier?

Context: Adding up accounts

- The second question can be imagined as borrowing twice at the same time:
- Borrow \$98.04 for one month, and pay back \$100 just like before
- Also borrow \$96.12 for two months, and pay back \$100

$$P = F/(1 + p)^T = \$100/1.02/1.02 = \$96.12$$

- So in total you borrow: \$194.16 and pay it back in two payments of \$100 each following month

Activity: More practice with short term installment loans

- On your own paper work out:
- How much can you borrow at 2% per month interest if you are willing to pay back \$100 each month for:
 - Three months
 - Six months
 - A year
- If you want to borrow \$500 now at 2% per month interest, how much do you need to pay back each month in order to be done in:
 - one month?
 - two months?
 - three months?

Activity proposed answers

- In principle the first is easy: just add it up!

	F	P	Total
1 month	\$100	$\$100/1.02^1 = \98.04	\$98.04
2 months	\$100	$\$100/1.02^2 = \96.12	\$194.16
3 months	\$100	$\$100/1.02^3 = \94.23	\$288.39
4 months	\$100	$\$100/1.02^4 = \92.38	\$380.77
5 months	\$100	$\$100/1.02^5 = \90.57	\$471.35
6 months	\$100	$\$100/1.02^6 = \88.80	\$560.14
...			
12 months	\$100	$\$100/1.02^{12} = \78.85	\$1057.53

- However, this gets a bit tedious. We will learn better today.

Activity second proposed answer

- \$500 now, pay it back in one month is easy:

$$P = \$500$$

$$F = ?$$

$$p = 0.02 \text{ per month}$$

$$T = 1 \text{ month}$$

- $F = P(1 + p)^T = \$500(1.02) = \510 in one payment, next month

- In two payments? Trickier. Remember our table?

\$100 each month got us \$194.16, but we need \$500

$$\text{So solve } \frac{\$500}{\$194.16} = \frac{?}{\$100}$$

- We pay $\$100(\$500)/(\$194.16) = \257.52 for two months
- Or $\$100(\$500)/(\$288.39) = \173.38 for three months

Fast: The formula

- $P = Mq \frac{1-q^T}{1-q}$

P is present value

M is periodic payment

p is periodic interest rate

T is number of periods

$q = 1/(1 + p)$ helps discount future payments into the present

- For example: $P = ?$, $M = 100$, $p = 0.02$, $T = 12$, $q = 1/1.02$, and

$$P = Mq \frac{1 - q^T}{1 - q} = \$100/1.02 * (1 - 1/1.02^{12}) / (1 - 1/1.02) = \$1057.53$$

Activity: More practice

- How much can you borrow at 2% per month interest if you are willing to pay back \$100 each month for:
 - Two years
 - Three years
 - Five years
 - 10 years
 - 20 years
 - FOREVER (bonus)

Second activity answers

- Two years: $\$100/1.02 * (1 - 1/1.02^{24})/(1 - 1/1.02) = \1891.39
- Three years: $\$100/1.02 * (1 - 1/1.02^{36})/(1 - 1/1.02) = \2548.88
- Five years: $\$100/1.02 * (1 - 1/1.02^{60})/(1 - 1/1.02) = \3476.09
- Ten years: $\$100/1.02 * (1 - 1/1.02^{120})/(1 - 1/1.02) = \4535.54
- 20 years: $\$100/1.02 * (1 - 1/1.02^{240})/(1 - 1/1.02) = \4956.86
- FOREVER: $\$100/1.02 * (1 - 1/1.02^{\infty})/(1 - 1/1.02) =$
 $\$100/1.02/(1 - 1/1.02) = \5000.00

Assignment and exit slip

- Good book problems #1-4, #19, #21, #37, #41, #43, #45, #47, #49
- Practice exam released on Tuesday, work it before Friday
- **Exit slip:** How much can you borrow today at 2% per month interest, if you are willing to pay back \$200 every month for a year?