

Name: _____

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MA111
Ch. 10 Exam (practice)
2012-10-08

Part I: Matching

- 1 Percentage increase formula
- 2 Compound interest formula
- 3 Installment loan formula
- 4 Present value of 20 years worth of monthly payments of \$100 at 5% monthly interest
- 5 Future value of \$100 after one period of 5% and three periods of 20% interest
- 6 Present value of three monthly payments of \$100 at 5% monthly interest
- 7 Future value of three monthly payments of \$100 at 5% monthly interest

- (1) $N = A(1 + p)$, N is new value, A is the old value, p is percentage as a decimal
- (2) $F = P(1 + p)^T$, F is future value, P is present value, p is periodic interest rate, T is number of periods
- (3) $P = Mq \frac{1 - q^T}{1 - q}$, P is present value, M is periodic payment, p is periodic interest rate, T is number of periods, $q = 1/(1 + p)$ helps discount future payments into the present
- (4) $\$100(1/1.05) \frac{1 - (1/1.05)^{240}}{1 - (1/1.05)}$
- (5) $\$100(1.05)(1.2)^3$
- (6) $\$100/(1.05) + \$100/(1.05)^2 + \$100/(1.05)^3$
- (7) $\$100(1.05)^2 + \$100(1.05) + \$100$

Percentage Increase

1. If \$300 is increased by 25%, what is the result?

$$\$300 (1.25) = \boxed{\$375}$$

or 25% of \$100 is \$25
25% of \$300 is \$75
so $\boxed{\$375}$ is $\$300 + \75

2. If \$300 is decreased by 12%, what is the result?

$$\$300 (1 - 0.12) = \$300 (0.88) = \boxed{\$264}$$

or 12% of \$300 is \$36, so $\$300 - \$36 = \boxed{\$264}$

3. If \$300 is increased by 10%, and the result is increased by 10%, what is the final result?

$$\$300 (1.1) (1.1) = \boxed{\$363}$$

or

$$\$300 \xrightarrow[\$30]{10\%} \$330 \xrightarrow[\$33]{10\%} \boxed{\$363}$$

4. If \$300 is increased by 2%, the result is decreased by 3%, and that result is increased by 4%, what is the final result?

$$\$300 (1 + 0.02) (1 - 0.03) (1 + 0.04)$$

$$= \$300 (1.02) (0.97) (1.04)$$

$$= \boxed{\$308.69}$$

5. Which is the smaller number: (a) \$300 or (b) the result of first increasing \$200 by 50%, and then decreasing the result by 50%?

(a) is \$300

$$(b) \text{ is } \$200 (1.5) (0.5) = \boxed{\$150}$$

b is smaller

↑
probably should
be \$300

$300 (1.5) (0.5)$
is 225, still smaller

Compound interest

1. How much does one pay back a year later, if one borrows \$300 at 1.5% per month interest?

$$P = 300$$

$$F = ?$$

$$r = 0.015 \text{ per month}$$

$$T = 12 \text{ months}$$

$$F = 300 (1.015)^{12} = \$358.69$$

2. How much can one borrow today at 1.5% per month interest, if one is willing to repay \$500 two years from now?

$$P = ?$$

$$F = 500$$

$$r = 0.015 \text{ per month}$$

$$T = 24 \text{ months}$$

$$P = 500 / (1.015)^{24} = \$349.77$$

3. If one borrows at 19% per month interest (crazy), how many months until the debt has doubled?

Months	Value
0	100% = 1
1	119% = 1.19
2	141.61% = $(1.19)^2$
3	168.516% = $(1.19)^3$
4	200.534% = $(1.19)^4$

4 months gives 200%, double.

(to be precise, very slightly less than 4 months)

4. If one borrows \$300 and repays \$336 a month later, what is the monthly interest rate?

$$P = 300$$

$$F = 336$$

$$r = ?$$

$$T = 1$$

$$336 = (1+r)^1 (300)$$

$$\frac{336}{300} = 1+r$$

$$1.12 = 1+r$$

$$0.12 = r$$

$$r = 12\% \text{ per month}$$

5. Which is the smaller number: (a) The amount to repay a \$300 debt a year later at 2% per month interest, (b) The amount to repay a \$300 debt a year later at 0.5% per week interest (assuming 52 weeks in a year)?

$$(a) \$300 (1.02)^{12} = \$380.47$$

$$(b) \$300 (1.005)^{52} = \$388.83$$

(a) is smaller

Amortized loans

1. How much do you owe after 6 months if you borrow \$300 at 1.3% per month interest and pay back \$50 at the end of every month (a total of \$300)?

Each New month is

$$(0.13) \cdot (1.013) - 50$$

\uparrow Plus Interest \uparrow Minus Payment

# Months	Debt
0	300.00
1	253.90
2	207.20
3	159.89
4	111.97
5	63.43
6	14.25

\$14.25 left after six months

2. How much can you borrow now at 1% per month if you are willing to pay back \$50 every month for a year?

$P = ?$
 $M = 50$ per month
 $r = 0.01$ per month
 $T = 12$ months
 $g = 1/1.01$

$$P = Mg(1-g^T)/(1-g) = 50(1/1.01)(1-1/1.01^{12})/(1-1/1.01)$$

$P = \$562.75$

3. How much should you pay back every month if you want to borrow \$500 now at 1% per month and be done paying it back after 7 months?

$P = 500$
 $M = ?$ per month
 $r = 0.01$ per month
 $T = 7$ months
 $g = 1/1.01$

$$500 = M(1/1.01)(1-1/1.01^7)/(1-1/1.01)$$

$$500 = M(6.72819)$$

$M = \$500/6.72819 = \74.31

4. Which is the smaller amount: (a) \$50 per month for a year, or (b) the monthly payment to repay a \$500 at 3% per month interest in a year.

(a) \$50 per month

(b) $500 / ((1/1.03)(1-1/1.03^{12}) / (1-1/1.03)) = \50.23 per month

(a) is slightly smaller