Name: __Kathryn__

Part I: Matching

_____1_____ Percentage increase formula

_____2_____ Compound interest formula

_____3_____ Installment loan formula

_____4_____ Present value of 20 years worth of monthly payments of $100 at 5% monthly interest

_____5_____ Future value of $100 after one period of 5% and three periods of 20% interest

_____6_____ Present value of three monthly payments of $100 at 5% monthly interest

_____7_____ Future value of three monthly payments of $100 at 5% monthly interest

(1) $N = A(1 + p)$, N is new value, A is the old value, p is percentage as a decimal

(2) $F = P(1 + p)^T$, F is future value, P is present value, p is periodic interest rate, T is number of periods

(3) $P = Mq\frac{1 - q^T}{1 - q}$, P is present value, M is periodic payment, p is periodic interest rate, T is number of periods, $q = 1/(1 + p)$ helps discount future payments into the present

(4) $\$100(1/1.05)\frac{1 - (1/1.05)^{240}}{1 - (1/1.05)}$

(5) $\$100(1.05)(1.2)^3$

(6) $\$100/(1.05) + \$100/(1.05)^2 + \$100/(1.05)^3$

(7) $\$100(1.05)^2 + \$100(1.05) + \$100$

Percentage Increase

1. If $300 is increased by 25%, what is the result?

$$N = A(1+p)$$
$$= 300(1+.25) = 300(1.25) = \boxed{\$375}$$

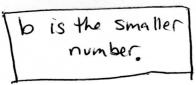2. If $300 is decreased by 12%, what is the result?

$$N = A(1+p)$$
$$= 300(1+{}^-.12) = 300(.88) = \boxed{\$264}$$

3. If $300 is increased by 10%, and the result is increased by 10%, what is the final result?

$$300(1+.1)(1+.1) = 300(1.1)(1.1) = \boxed{\$363}$$

4. If $300 is increased by 2%, the result is decreased by 3%, and that result is increased by 4%, what is the final result?

$$300(1+.02)(1-.03)(1+.04)$$
$$= 300(1.02)(.97)(1.04) = \boxed{\$308.69}$$

5. Which is the smaller number: (a) $300 or (b) the result of first increasing $200 by 50%, and then decreasing the result by 50%?

$$a = \$300$$
$$b = 300\,200(1+.5)(1-.5)$$
$$= 200(1.5)(.5) = \$150$$

$\boxed{\text{b is the smaller number.}}$

Compound interest

1. How much does one pay back a year later, if one borrows $300 at 1.5% per month interest?

$$F = P(1+p)^T \qquad T = 12 \text{ months}$$
$$= 300(1+.015)^{12} = 300(1.015)^{12} \approx \boxed{\$358.68}$$

2. How much can one borrow today at 1.5% per month interest, if one is willing to repay $500 two years from now?

$$F = P(1+p)^T \qquad T = 24 \text{ months}$$
$$500 = P(1+.015)^{24}$$
$$= P(1.015)^{24}$$
$$\approx P(1.4295)$$
$$P = \frac{500}{1.4295} \approx \boxed{349.77}$$

3. If one borrows at 19% per month interest (crazy), how many months until the debt has doubled?

$$F = P(1+p)^T \qquad \text{When the debt has doubled, } F = 2P$$
$$2P = P(1+p)^T$$
$$2 = (1+.19)^T$$
$$2 = (1.19)^T$$

$$\ln 2 = \ln(1.19)^T$$
$$\ln 2 = T(\ln(1.19))$$
$$\frac{\ln 2}{\ln(1.19)} = T$$

$$\boxed{T \approx 3.98}$$
The debt has doubled after about 4 months!!

4. If one borrows $300 and repays $336 a month later, what is the monthly interest rate?

$$F = P(1+p)^T \qquad T = 1 \text{ month}$$
$$336 = 300(1+p)^1$$
$$\frac{336}{300} = 1+p$$
$$1.12 = 1+p$$

$$p = .12 \text{, so } \boxed{\text{the monthly interest is 12%}}$$

5. Which is the smaller number: (a) The amount to repay a $300 debt a year later at 2% per month interest, (b) The amount to repay a $300 debt a year later at 0.5% per week interest (assuming 52 weeks in a year)?

(a) $F = P(1+p)^T \qquad T = 12 \text{ months}$
$$= 300(1+.02)^{12}$$
$$= 300(1.02)^{12} \approx \$380.47$$

(b) $F = P(1+p)^T \qquad T = 52 \text{ weeks}$
$$= 300(1+.005)^{52}$$
$$= 300(1.005)^{52} = \$388.83$$

$$\boxed{\text{(a) is the smaller number}}$$

Amortized loans

1. How much do you owe after 6 months if you borrow \$300 at 1.3% per month interest and pay back \$50 at the end of every month (a total of \$300)?

$$P = Mq\frac{1-q^T}{1-q} \qquad q = \frac{1}{1+p} = \frac{1}{1.013}$$

This would have paid it off if we had only borrowed \$286.81, but we borrowed more.

$$= 50q\frac{(1-q^6)}{(1-q)} = \$286.81$$

$$300 - 286.81 = 13.19$$

We can just consider that amount separately compounded monthly.

We still owe $(13.19)(1.013)^6 = \boxed{\$14.25}$

2. How much can you borrow now at 1% per month if you are willing to pay back \$50 every month for a year?

$$P = Mq\frac{1-q^T}{1-q} \qquad q = \frac{1}{1+.01} \qquad T = 12 \text{ months} \qquad P = ?$$

$$= 50q\frac{(1-q^{12})}{1-q} = \boxed{\$562.75}$$

3. How much should you pay back every month if you want to borrow \$500 now at 1% per month and be done paying it back after 7 months?

$$P = Mq\frac{1-q^T}{1-q} \qquad q = \frac{1}{1+.01} \qquad T = 7 \text{ months} \qquad M = ?$$

$$500 = Mq\frac{(1-q^7)}{1-q} \qquad M = \frac{500}{6.72819} \boxed{\$= 74.31}$$

$$500 = M(6.72819..)$$

4. Which is the smaller amount: (a) \$50 per month for a year, or (b) the monthly payment to repay a \$500 at 3% per month interest in a year.

$$T = 12 \text{ months}$$

Which is smaller per month?

(a) \$50

(b) $P = Mq\frac{(1-q^T)}{1-q} \qquad q = \frac{1}{1+.03} \qquad M = ?$

$\boxed{\text{(a) is smaller}}$

$$500 = Mq\frac{(1-q^{12})}{1-q}$$

$$\frac{500}{\left(q\frac{(1-q^{12})}{1-q}\right)} = M$$

$$50.23 = $$