Practice Exam

Name: JACK SCHMIDT

MA111
2012-10-26

Instructions: You may not use any network-capable device, including cell-phones. You are graded on how well you communicate your understanding on this exam. You must show your work. Clearly indicate paths. Work that cannot be read is not worth credit.

Part I: Vocabulary

Match the word with its definition:

A	Graph	H	Euler Path
B	Vertex Set	I	Euler Circuit
C	Edge Set	J	Exhaustive Route
D	Degree	K	Eulerization
E	Path	L	Optimal Exhaustive Route
F	Circuit	M	Optimal Eulerization
G	Connected	N	Handshaking Lemma

(A) A collection of relationships with two parts: a vertex set and an edge set

(B) The list of vertices of a graph, or at least a way to tell exactly what the vertices of a graph are

(C) The list of edges of a graph, or at least a way to tell how many relationships are between any two vertices

(D) The number of times a vertex appears in the edge set; the number of edges adjacent to a vertex (where loops count twice)

(E) A sequence of edges, each adjacent to the next, that start and stop at different vertices

(F) A sequence of edges, each adjacent to the next, that start and stop at the same vertex

(G) A graph such that between any two distinct vertices there is a path

(H) A path using all the edges in a graph exactly once

(I) A circuit using all the edges in a graph exactly once

(J) A circuit using all the edges in a graph at least once

(K) A list of the edges to be used more than once in an exhaustive route

(L) An exhaustive route of shortest possible length

(M) An Eulerization of an optimal exhaustive route

(N) The total degree is twice the number of edges

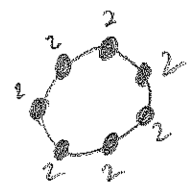
Part II: Handshaking lemma and graph reconstruction

1. Construct a graph with vertices of degree 1, 1, 1, 1, 1, 1, 1 (that is seven 1s) or explain why no such graph exists.

Total Degree = 1+1+1+1+1+1+1 = 7
#Edges = 7/2 = 3.5
Crazy! Impossible by Handshaking Lemma.

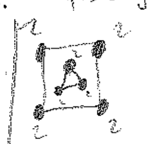
2. Construct a graph with vertices of degree 2, 2, 2, 2, 2, 2, 2 (that is seven 2s) or explain why no such graph exists.

Total Degree = (2)(7) = 14
#Edges = 14/2 = 7



7 edges

4+3 edges

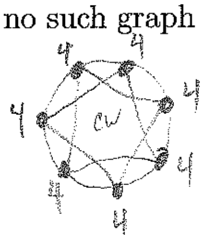


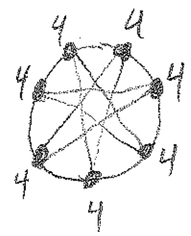
Not Connected but OK

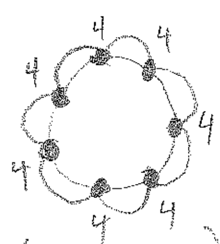
3. Construct a graph with vertices of degree 3, 3, 3, 3, 3, 3, 3 (that is seven 3s) or explain why no such graph exists.

Total Degree = (3)(7) = 21
#Edges = 21/2 = 10.5, half Edge? Crazy!
Impossible by Handshaking Lemma

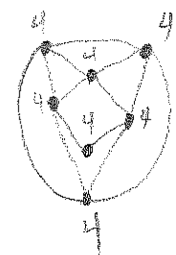
4. Construct a graph with vertices of degree 4, 4, 4, 4, 4, 4, 4 (that is seven 4s) or explain why no such graph exists.



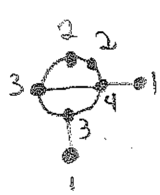


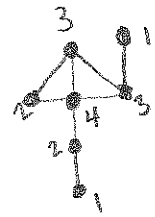


(Double Edges)



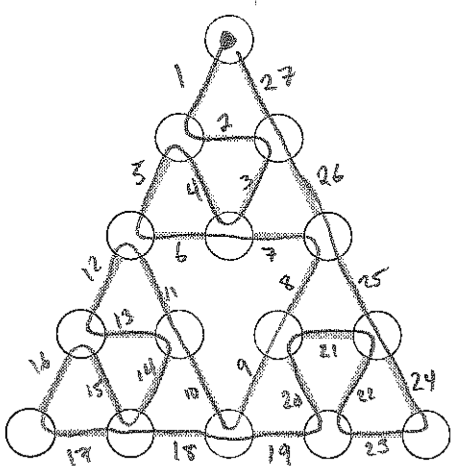
5. Construct a graph with vertices of degree 1, 1, 2, 2, 3, 3, 4 or explain why no such graph exists.

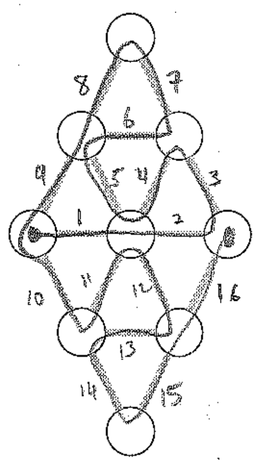


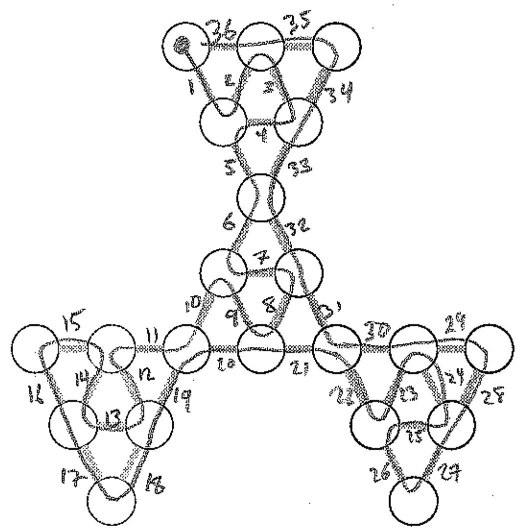


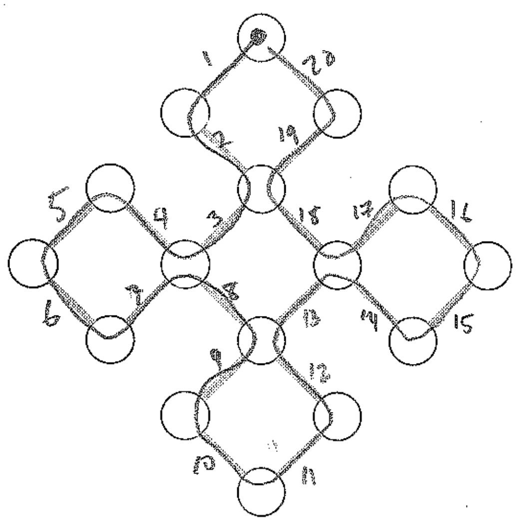
Part III: Finding the Euler circuits

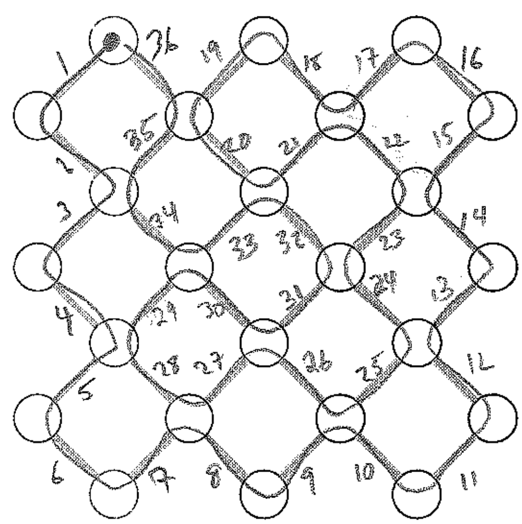
1. For each graph label the edges 1, 2, 3, ... in order of an Euler circuit or Euler path. You must clearly indicate the turns taken at each vertex. Do not make any right angle turns inside a circle.











Part V: No, really, find them!

Label the degrees of each vertex, and then find optimal Eulerizations. Describe the Eulerization by darkening the edges that are repeated (don't add any truly new edges, only repeat old ones, until all the degrees are even). Each of the graphs is connected.