

# MA111: Contemporary mathematics

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Entrance Slip (due 5 min past the hour):

- Three friends buy a DVD 3-pack, each paying \$8.
- They each think they paid for different movies:

	Adam	Bart	Chad
Momma Mia!	\$ 6	\$ 6	\$6
Kung Fu Panda	\$12	\$ 6	\$9
WALL-E	\$ 6	\$12	\$9

- The friends are leaving the store, who should get which movie?

Today we investigate fair division. Exam is Nov 19. HW is due Nov 19.

## Context: How to share?

- The entrance slip has some trouble:

somebody gets the bad piece

- We'd like to say it is not fair

He paid \$8 but only got \$6 worth of movie

- What can we do to even things out?

- Our basic problem is who gets what

and do we need to compensate anyone with money

## Activity: More than one way to be fair

- The worksheet describes several different divisions
- Each one has some sort of claim to “fair”
- Calculate how good each person feels
- Try to decide what is important in dividing the stuff up
- For the last question, what is the right answer?

## Fast: Fair division

- Fair division is a social choice problem
- Three main ingredients to the problem:
  - The people
  - The loot
  - How the people value the loot
- First we talk about solutions to particular fair division problems
- Then we talk about games that lead to solutions

## Fast: A fair division

- Each person must receive as much value as they paid for
- A division is then called **fair** if every person thinks they get at least their fair share
- We've seen there can be more than one way to get a fair division
- In #2, everyone thinks they got exactly their fair share
- In #3, if Adam and Bart pay Chad \$2 each, then everyone paid \$8 and got \$10 worth of stuff!

## Fast: utility function

- People value the same good differently
- Each person has a **utility function** that takes the booty and values it in dollars

Adam's utility function (his heart, or cinematic tastes) told him MM was worth \$6, KFP was worth \$12, and WE was worth \$6

- Our utility functions in this course are “additive”

The three movies together are worth  $\$6 + \$6 + \$12 = \$24$  to Adam

## Fast: utility functions can make happiness

- Since Bart and Adam disagree on the value of the two movies we can make happiness out of nothing
- If Adam gets his \$12 movie, and Bart gets his \$12 movie, and Chad gets his \$6 movie, then we have \$30 total worth of movie, but they paid only \$24 total
- So we have a chance at a happy ending!
- But how do we get people to agree to the happy ending?
- We will propose very carefully constructed games

## Fast: fair division games

- We need games with very special properties:
- The rules are simple and observable; if we video tape the game being played, everyone can tell whether anyone cheated
- There is a winning strategy; the winning strategy is easy to memorize
- Once we have such games, then people can agree to play them
- They are guaranteed to win unless someone obviously cheats, and then everyone will know that person stole
- The next few classes we will cover specific games



# Assignments and exit slip

- Read 3.1 and 3.2 in the book.
- Start on online homework; 21 problems due Nov 19
- **Exit slip:** What is wrong with the following game:
  - ① Adam chooses his favorite movie
  - ② Bart chooses his favorite movie from among those left
  - ③ Chad takes the left-over movie and tells Adam and Bart how much they owe him