MA111: Contemporary mathematics

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Entrance Slip (due 5 min past the hour):

- You and the grader are playing a game to divide 10 points.
- You write down a number between 0 and 10 on your index card.
- The grader decides whether you get that many points,
  or whether she does.
- The other person gets the rest of the points.

Today we investigate two games. Written Project is due Nov 9. Exam and HW is Nov 19.

Context: Our first game and a second game

- The entrance slip game has different strategies

- You could hope the grader does not want points and write "10"

- But if she does, then that leaves you with 0. Oops.

- A safe strategy is to answer 5. Either you get 5 or $10 - 5 = 5$.

- Let's look another game played in a creepy old forest

Activity: Plan your strategy

- Two players and Sweeny; a twenty dollar bill and a hat

- Sweeny suggests a game to divide the twenty dollar bill

- Players take turns placing dollar bills in the hat

- Last player to put a dollar bill in the hat keeps the twenty

- Sweeny keeps the hat and the dollar bills

- What is your strategy? Who wins?

Activity recap: The only way to win is not to play

- If you look only at your own strategy it is easy:

- On your turn, you can put in a dollar to gain $20 (so +$19)

- Or you can fold, and not gain anything

- Clearly you should put in the dollar

- But so should the other player!

- When does it stop?

- Only when one of you runs out of money.

- That person loses all their money, the other loses the same amount but gets a twenty

Fast: games to divide fairly

- The activity game is evil, but is used on many reality shows

- The entrance slip game is good, and has been used for millenia

- **You cut, I choose** is very simple:

    1. One player divides the loot into two piles

    2. The other player chooses a pile to keep

    3. First player keeps the other

- Notice the rules are simple and observable

Fast: How do you win the game?

- If you go first, then it is like the entrance slip

- Does she want the points? If she does, and you say 10, then oops.

- The only safe strategy is to divide it evenly and say 5

- Key: but this strategy is safe!

- There is nothing (legal) the other player can do to cheat you out of your fair share of half!

- If you don't divide it in half, you can lose.

Fast: How do you win as second player?

- Second player is even easier.

- There is a formula:

$$P = Mq\frac{1 - q^T}{1 - q} \xrightarrow{\nabla^2} \int_0^\infty e^{-\mu(t)^2} \, dt$$

where M is the monthly payment, q is the periodic discount rate, and μ is the utility function

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- Just kidding. You take the bigger piece.

- Seriously. Which piece is better? Ok, take that one.

- Seriously. You cannot lose unless you choose to lose.

- Simple rules, simple strategy.

Fast: Summary

- Simple rules: You cut, I choose (You keep the rest).

- Simple strategy: (1st) cut it in half. (2nd) take the bigger half.

- There are however some subtle points:

  - Who can tell whether the first player followed the strategy?

  - Who can tell whether the second player followed the strategy?

  - Which player would you rather be?

Assignment and exit slip

- Much of the rest of the chapter is concerned with how to (a) allow more than two players, or (b) make sure no player has a real advantage.

- Read 5.1, 5.2, 5.3; Do HW 1-3. Skim the rest.

- **Exit slip:** You and the professor are playing a game to divide 10 points between your exit slip grade and his personal point supply. He picks a number between 0 and 10. You either take that many points, or take the rest. Indicate which on your exit slip.

- The professor picks … 6.

- Make sure to show your work, including formulas.