MA111: Contemporary mathematics

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Entrance Slip (due 5 min past the hour):

- 3 friends chip-in $12 each on a half-strawberry half-chocolate cake.
- One friend cuts the cake into three pieces.
- The other two friends take which piece they want.
- What can go wrong?

Today we investigate three player lone divider.

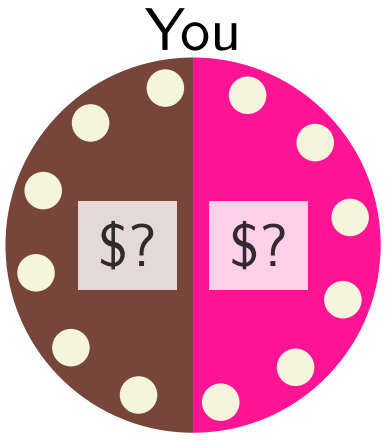
Written Project is due Nov 9. Exam and HW is Nov 19.

Context: How to make "You cut, I choose" work for 3

- If the divider is psychic, he can make three pieces:
  1. All of which are good for him
  2. Two of which are good for the other two people
- But people are (generally) neither psychic nor nice
- How do we set up the game so that anyone can divide?
- Let's have three volunteers come up so we can see

Activity: Lone divider

- Secretly write down a utility function for cake:



- In groups of 3 or 4, play this game:
  1. One player divides the cake into enough pieces; he makes the claim that each of the pieces is good enough for him
  2. Each player secretly marks which pieces they are willing to take
  3. Each player reveals which pieces they are willing to take
  4. Each player gets a piece they were willing to take, unless...
- Figure out what can go wrong on the 4th step
- What do we do to fix it?
- Feel free to play a few times
  (have different people divide; divide in a silly way)

Activity recap

- On the 4th step, everybody might only want one piece
- Well not EVERYBODY. The divider always gets a piece, right?
- And we can always give him a piece no-one will argue about, since the divider likes all pieces.
- So now we have one fewer player and one fewer piece, but the missing piece was a “bad piece” according to everyone
- So we actually have MORE cake per person now!
- So we play again, with fewer people and cake per person

Fast: Lone divider

- Requirements: Any number N of players; loot that can be divided arbitrarily and recombined without loss of value
- Rules:
  1. One player chosen at random is the divider. He divides the loot into N pieces. He declares that he will be happy with any of the N pieces.
  2. All other players write down which pieces they will be happy with.
  3. All players reveal which pieces.
  4. The divider is given a piece no-one is fighting over. Anybody else who can be given a piece without fighting is given a piece.
  5. Now play again with the remaining players and remaining pieces.
- 4 is called a "matching problem" (this year's Nobel prize was awarded for solving version of this that work for thousands or hundreds of thousands of people)
- 5 is called "induction" and is wonderfully freeing. We don't need to solve all the world's problems. Just one at a time.

Fast: strategy for divider

- Divide fairly in your own estimation, ignore everyone else
- Guaranteed to be “proportional” (fair)
- Every piece is exactly fair, and you get one of them
- Nobody else’s strategy can affect you!
- Only sad part: you never get anything extra

Fast: strategy for choosers

- Play honestly: anything that is at least your fair share, be willing to take; ignore everyone else
- If you are not in a fight, you get a fair piece
- If you are in a fight, no-one leaves with a piece you like
- In the fight case: everybody who leaves, takes a bad piece, leaving more for you!
- No matter what other people do, you get a fair piece in the first case, and more than fair in the second!
- If you lie, and only pick your favorite piece, then someone can leave with too much, and the next round you lose.

Assignment and exit slip

- Read and understand 5.1-5.3. Skim the rest of the chapter, especially 5.4.
- Project due on Friday. Paper version in class, electronic by 5pm, no exceptions.
- Homework due Nov 19.
- **Exit slip:** You are a chooser in Lone Divider.

  There are four players.

  There are four pieces: 5%, 30%, 30%, and 35%
- Which ones are at least fair to you?
- What can go wrong if you only choose the best piece?