# MA111: Contemporary mathematics

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## Entrance Slip (due 5 min past the hour):

- The Archduke of Lexington passed away, leaving his two children Duchess Lexi and Duke Ngton as heirs.

  The recent financial crisis has reduced his estate to $36 in cake.

  Lexi and Ngton divide the cake in such a way that both feel fairly treated.

  However, the Earl of Richmond reveals he is the son of the Archduke by a previous marriage and demands his third of the cake! The courts approve his claim, and ask Lexi and Ngton to give up the Earl's share.

- How much of Lexi's cake should the Earl get?

  How much of Ngton's cake should the Earl get?

## Today we investigate three player lone chooser.

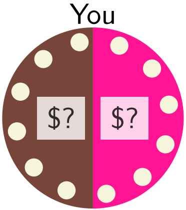
Written Project is due now. Exam and HW is Nov 19.

# Context: How to add a player to fair division

- We can use a fair division to make a new fair division
- We first fairly divide it amongst two of the players, A and B
- Then we divide it among three players using a simple idea:
- C gets $1/3$ of A's piece, and C gets $1/3$ of B's piece
- A and B's pieces added up to the whole, so $1/3$ of each adds up to $1/3$ of the whole
- We could even have a player D: D gets $1/4$ of A, B, and C's pieces.

# Activity: Lone chooser

- Secretly write down a utility function for cake:




- In groups of 3 or 4, play this game:

  1. Two of you (A and B) divide the cake fairly between the two of you
  2. Now A and B each split their piece into three pieces
  3. C chooses one of A's three pieces as their own,
     and one of B's three pieces as their own
  4. Now A, B, and C split their pieces into four pieces each
  5. D chooses one of each of their pieces as his own

- Play a few times to get used to calculating values

# Activity recap

- Theoretically NOTHING goes wrong!
- This method avoids the weird matching problem
- Unfortunately, it takes a long time with more than 3 or 4 people
- With 10 people, 9 people have to divide their slices into 10 sub-slices
- But for those 9 people, 8 people had to divide their slices into 9 sub-slices
- And so on.
- However, for three people it is fast and simple.

# Fast: Lone chooser

- Requirements: Any number $N$ of players; loot that can be divided arbitrarily and recombined without loss of value
- Rules:
  1. Choose one player to be left out; everyone else plays a game to fairly divide the loot (in a temporary way)
  2. Now each player with loot divides it into $N$ piles. They declare they would be happy losing any one of the piles.
  3. The left out player now selects one pile from each player with loot, and declares he would be happy with all those piles.
  4. The left out player gets all those piles, and everybody else keeps the rest of their loot.
- 1 is called "induction" and is used to make our game easier to start. Last class we used induction to make the game easier to finish.
- The math at work here is called the distributive property. It is nice that it can be used to distribute goods fairly.

# Fast: strategy for dividers

- Divide fairly in your own estimation, ignore everyone else.
- Guaranteed to be "proportional" (fair)
- Every piece is exactly fair, and you only lose one of them
- Nobody else's strategy can affect you!
- Only sad part: you always lose exactly $1/N$th of your extra loot.

# Fast: strategy for the chooser

- The strategy is given by a simple formula:

$$
\begin{aligned}
x = &-\frac{b}{3a} \\
&-\frac{1}{3a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d + \sqrt{\left(2b^3 - 9abc + 27a^2d\right)^2 - 4\left(b^2 - 3ac\right)^3}\right]} \\
&-\frac{1}{3a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d - \sqrt{\left(2b^3 - 9abc + 27a^2d\right)^2 - 4\left(b^2 - 3ac\right)^3}\right]}
\end{aligned}
$$

where $a$ is the size of $A$s slice, $b$ is the size of $B$s slice, $c$ is the average happiness of $A$ and $B$, and $d$ is the disadvantage incurred from choosing the piece they want you to choose.

# Fast: strategy for the chooser

- The strategy is given by a simple formula:

$$x = -\frac{b}{3a} - \frac{1}{3a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d + \sqrt{\left(2b^3 - 9abc + 27a^2d\right)^2 - 4\left(b^2 - 3ac\right)^3}\right]} - \frac{1}{3a}\sqrt[3]{\frac{1}{2}\left[2b^3 - 9abc + 27a^2d - \sqrt{\left(2b^3 - 9abc + 27a^2d\right)^2 - 4\left(b^2 - 3ac\right)^3}\right]}$$

  where $a$ is the size of $A$s slice, $b$ is the size of $B$s slice, $c$ is the average happiness of $A$ and $B$, and $d$ is the disadvantage incurred from choosing the piece they want you to choose.

- Just kidding.

# Fast: strategy for chooser

- Play honestly: take the best piece!
- Even if one player does not have much cake to begin with, you still get his best piece, so more than $\frac{1}{N}$th of his loot
- All the players combined have all the loot combined, so you get more than $\frac{1}{N}$th of all the loot combined
- No matter what other people do, you get at least a fair piece, usually more

# Assignment and exit slip

- Read and understand 3.1-3.4. Skim 3.5.
- Project due on Blackboard by 5pm. Paper version required now.
- Homework due Nov 19.
- **Exit slip:** You are a divider in Lone Chooser.

  There are four dividers and one chooser.

  You need to divide your loot into piles.
- What percentage of your loot should be in each pile?

  If there are multiple piles, give the percentage for each.

  For instance, if you need to make two piles of sizes 36% and 64% then say "Two piles of 36% and 64%"