DEPARTMENT OF MATHEMATICS

Ma 162 Final Exam -5 December 15, 2009

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.

Instructions: Be sure that your name, section number, and student ID are filled in below. Cell phones must be OFF and put away before you open this exam. You may use calculators (including graphing calculators, but no laptops or cellphone calculators) for checking numerical calculations, but you must show your work to receive credit.
Put your answers in the answer boxes provided, and show your work.
If your answer is not in the box or if you have no work to support your answer, you will receive no credit.
The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	Maximum Score	Actual Score
1	15	
2	15	
3	8	
4	10	
5	16	
6	12	
7	12	
8	12	
Total	100	

Please fill in the information below.

NAME: _____ **Section:** _____

SECTION NO: _____

1. Set this problem up, by stating the chosen variables, the function to be maximized and **all** the inequalities. **Do not solve the problem.**

   "The Gates" company makes two types of gates called "Royal" and "Estate".

   Each Royal gate sells for $ 170, requires 20.0 pounds of iron and 2.0 hours of labor.

   Each Estate gate sells for $ 90, requires 13.0 pounds of iron and 5.0 hours of labor.

   The company has 300 pounds of iron on hand and 330 hours of labor available.

   Set up a LPP whose solution will determine how many gates of each type should be produced to maximize the company's profit.

   i) Define and explain all the variables you use.

Variables are:

   ii) Now describe the LPP explicitly by writing **all the inequalities** and specifying the objective function.

Maximize: Profit $P =$ Subject to:

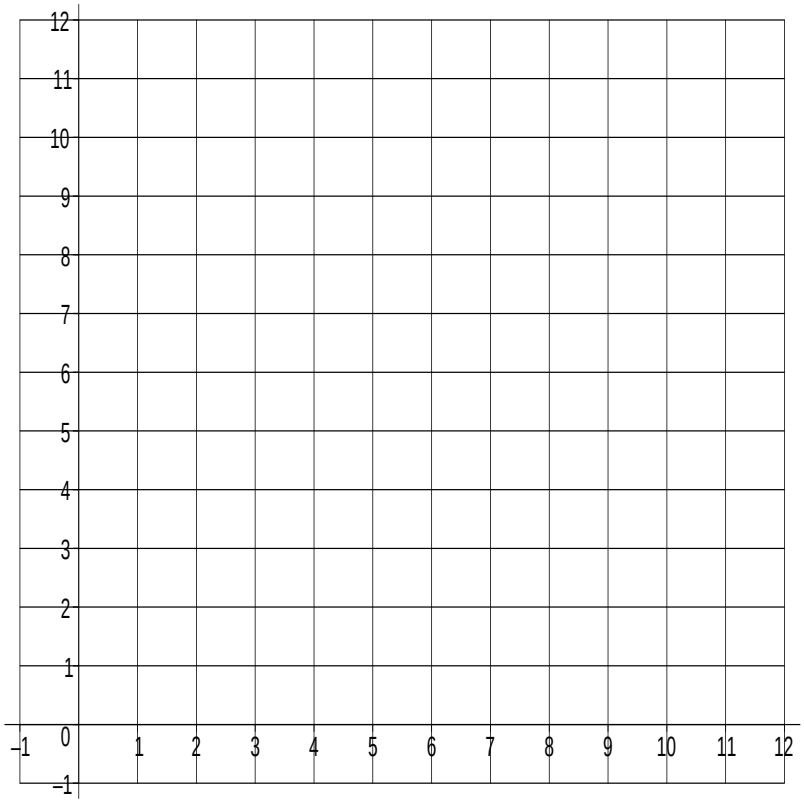
   iii) The initial Simplex tableau will be:

2. i) Sketch and shade the region described by the inequalities. Compute the coordinates of the corner points and mark them on your graph.
$2\,x + 3\,y \leq 18$

$y \leq x - 2$

$x \geq 0$

$y \geq 0$



ii) Find the maximum value of the function, $P = 8\,x + 14\,y$ on the region.

Answer: $P =$ [ ] at $x =$ [ ], $y =$ [ ].

3. Here is a final tableau associated with a maximal LPP.

x	y	z	s	t	u	P	constants
-2	0	1	0	-4	-13	0	6
-1	1	0	0	0	1	0	4
1	0	0	1	1	4	0	2
6	0	0	0	5	25	1	15

(a) Using your knowledge of the Simplex algorithm, determine the solution to the maximal LPP.

Value of P= ☐ (x , y , z)=(☐ , ☐ , ☐)

(b) Recall that the solution of the **dual minimization problem** can be read from the same tableau. Determine the solution of the dual minimization LPP.

Value of C= ☐ (s , t , u)=(☐ , ☐ , ☐)

4. Set this problem up, by stating the chosen variables, the equations to be solved and **the initial augmented matrix. Do not solve the problem.**

   The "MOMS" company supplies carepacks to mothers who want to send the packs to their kids living in hostels. There are three different types of packs called "Fun", "Joy" and "Cheer".

   The "Fun" pack contains 18 cookie bags and 21 chip bags.

   The "Joy" pack contains 23 cookie bags and 17 chip bags.

   The "Cheer" pack contains 16 cookie bags and 21 chip bags.

   The manager tells John to make 380 care packs using the available 7300 cookie bags and 7420 chips bags.

   John asks how many of each type is he supposed to make, but the manager tells him that he has studied linear algebra and should write down and solve equations!

   But John has forgotten how to do this. Help John figure this out by setting up equations for him and writing the initial augmented matrix.

   Use x,y,z to denote the number of "Fun", "Joy" and"Cheer" carepacks respectively.

The equations to be solved are:

The augmented matrix is:

5. (i) Consider the following system of linear equations.

$$x + 2\,y + 4\,z = 2$$
$$3\,x + 7\,y + 14\,z = 10$$

Write down the augmented matrix for this system of equations.

Reduce the augmented matrix to REF (the row echelon form). It is essential to show the steps of row reductions and explicitly write the row operations used. You may need more or fewer boxes than provided. Use the empty space on the back of the previous page if you need additional space.

(ii) Using above calculations, determine all the solutions to the system of equations in x, y, z given above.

6. A group of 141 adults were asked if they visit fast-food restaurants or sit-down restaurants.

   It was found that 76 adults visit fast-food restaurants and 61 visit sit-down restaurants; while 20 adults announced that they visit neither since they don't like to eat out.

   (i) Estimate the probability that a random adult likes to eat out- i.e. eats at a fast-food restaurant or a sit-down restaurant (or both). ☐

   (ii) Estimate the probability that a random adult likes both fast-food and sit-down restaurants. ☐

   (iii) Estimate the probability that a random adult eats at a fast-food restaurant but not at a sit-down restaurant. ☐

7. An experiment consists of casting a die and observing the number on top. Let A be the event that the number on top is 1 or 3 or 5 or 6. Let B be the event that the number on top is less than or equal to 4.

   It is observed by experimentation that $P(A) = 55\%$ and $P(B) = 55\%$

   Answer the following questions **based on these experimental observations.**

   (i) What is the probability that the number on top is 1 or 3 ? $\boxed{}\%$.

   (ii) What is the probability that the number on top is 2 or 4 ? $\boxed{}\%$.

   (iii) The experimenter concludes that this die must be loaded since a fair die would show the numbers $2, 4$ with probability equal to $\boxed{}\%$.

8. Two fair dice are tossed, one red and one green.

Let A be the event that the sum of the numbers on top is 6.

Let B be the event that the number on top of the red die is **not** 1. Answer the following:

(i) What is the probability that the sum of the numbers on top is 6, i.e. what is $P(A)$?

(ii) the number on top of the red die is **not** 1, i.e. what is $P(B)$?

(iii) What is the probability that the sum of the numbers on top is 6, given that the number on top of the red die is **not** 1, i.e. what is $P(A|B)$?

(iv) What is $P(A \bigcap B)$?

(v) From the above calculations, would you consider A and B independent events? Why?

1 Answer Key for exam4v-5

1. ◇ $P = 170\,x + 90\,y \quad 0 \le x \quad 0 \le y \quad 20\,x + 13\,y \le 300 \quad 2\,x + 5\,y \le 330$

2. ◇ $P = \frac{388}{5}$ at $x = \frac{24}{5}$ $y = \frac{14}{5}$.

3. ◇ (a) P= 15 (x,y,z) $[0, 4, 6]$
   ◇ (b) C= 15 (s,t,u) $[0, 5, 25]$

4. ◇
$$\begin{aligned} x + y + z &= 380 \\ 18\,x + 23\,y + 16\,z &= 7300 \\ 21\,x + 17\,y + 21\,z &= 7420 \end{aligned}$$

5. ◇
$$\begin{array}{cccccccc} 1 & 2 & 4 & 2 & 1 & 0 & 0 & -6 \\ 3 & 7 & 14 & 10 & 0 & 1 & 2 & 4 \end{array} \quad [x, y, z] = [-6, -2\,z + 4, z]$$

6. ◇ (i) $[\frac{121}{141}$, "85.82", "*percent*"] (ii) $[\frac{16}{141}$, "11.35", "*percent*"] (iii) $[\frac{20}{47}$, "42.55", "*percent*"]

7. ◇ (i) 10 %(ii) 45 %(iii) 33.0 %

8. ◇ (i) 0.1389 (ii) 0.8333 (iii) 0.1333 (iv) 0.11107889 (v)Yes $[0.1157, 0.1111]$