MA162: Finite mathematics

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University of Kentucky

September 26, 2012

Schedule:

- HW 2.6, 3.1 are due Friday, Sep 28th, 2012
- HW 3.2, 3.3 are due Friday, Oct 5th, 2012
- HW 4.1, 4.2 are due Friday, Oct 12th, 2012
- Exam 2 is Monday, Oct 15th, 5:00pm-7:00pm in BS107 and BS116.
- Exam grades not available until Monday; will be announced when ready

Today we will cover 3.1: graphing linear inequalities

Exam 2: Overview

- Ch. 3, Linear optimization with 2 variables
  1. Graphing linear inequalities
  2. Setting up linear programming problems
  3. Method of corners to find optimum values of linear objectives
- Ch. 4, Linear optimization with millions of variables
  1. Slack variables give us flexibility in RREF
  2. Some RREFs are better (business decisions) than others
  3. Simplex algorithm to find the best one using row ops
  4. Accountants and entrepreneurs are two sides of the same coin

3.1: Inequalities

- Xylophones cost \$200 each and Yukuleles cost \$100 each
- Your need instruments for your new band Glük-N-Spiel
- Your insane and rich uncle only gave you a budget of \$1000
- What are your options?

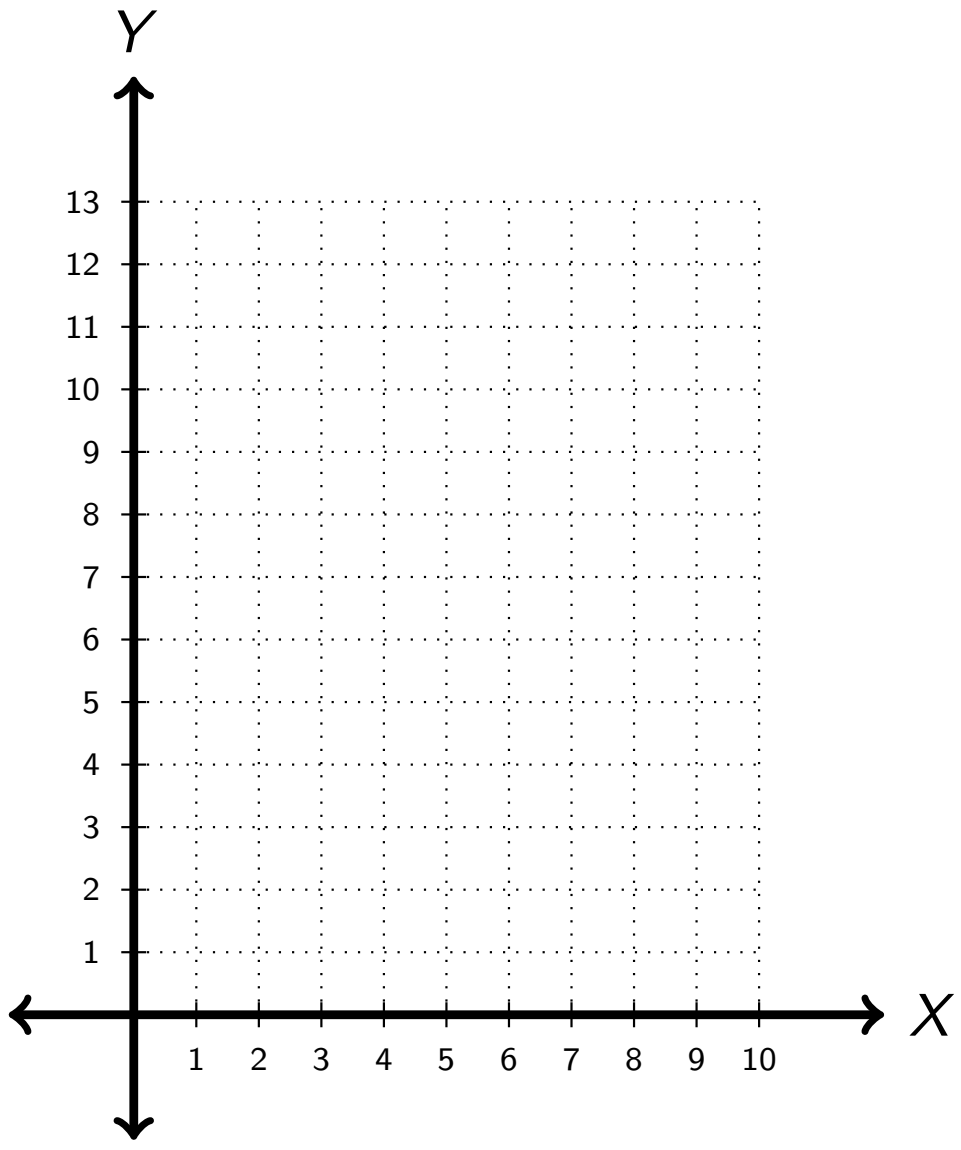
$$200x + 100y = 1000$$

3.1: Inequalities

- Xylophones cost \$200 each and Yukuleles cost \$100 each
- Your need instruments for your new band Glük-N-Spiel
- Your insane and rich uncle only gave you a budget of \$1000
- What are your options? Don't have to spend it all!

$$200x + 100y \leq 1000$$

3.1: Graphing inequalities



$200x + 100y = 1000$

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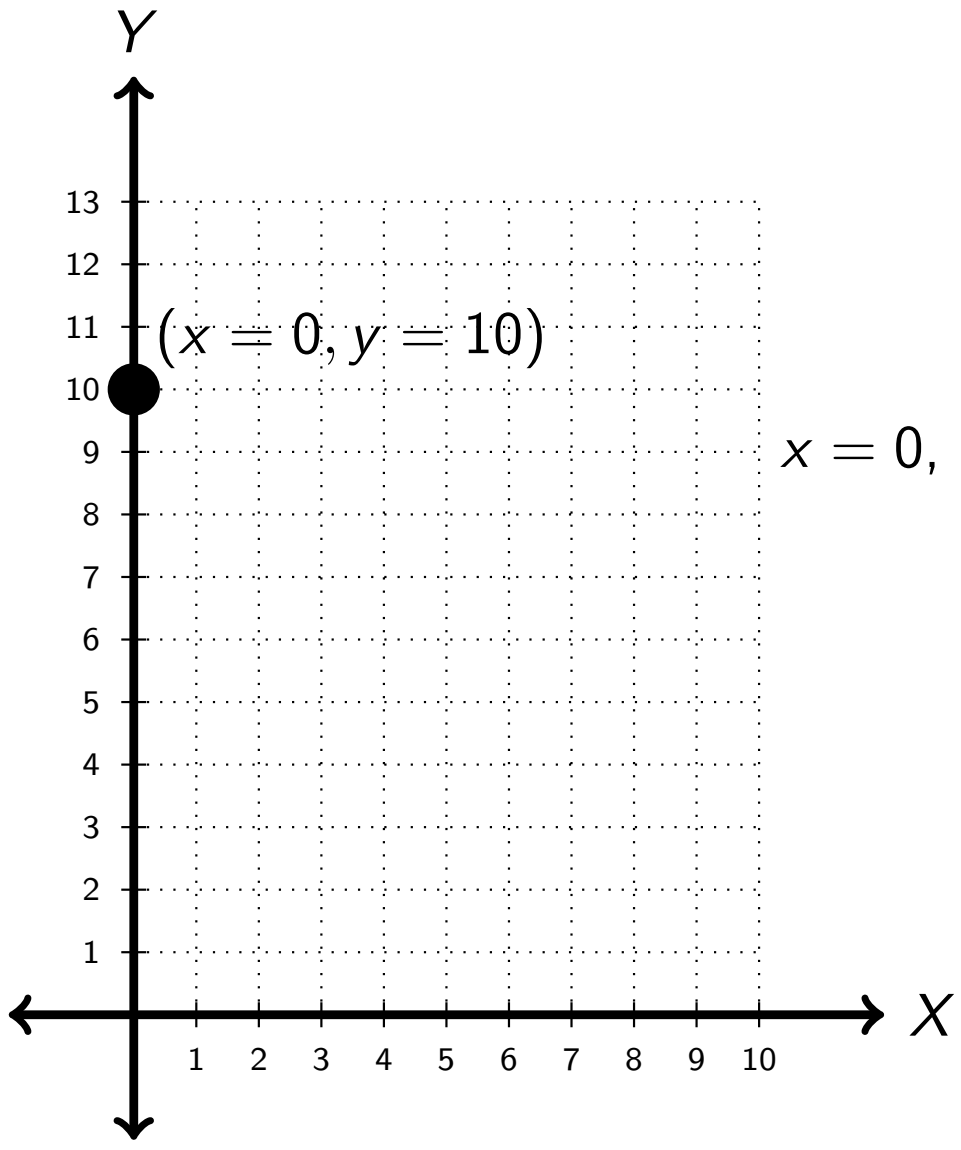
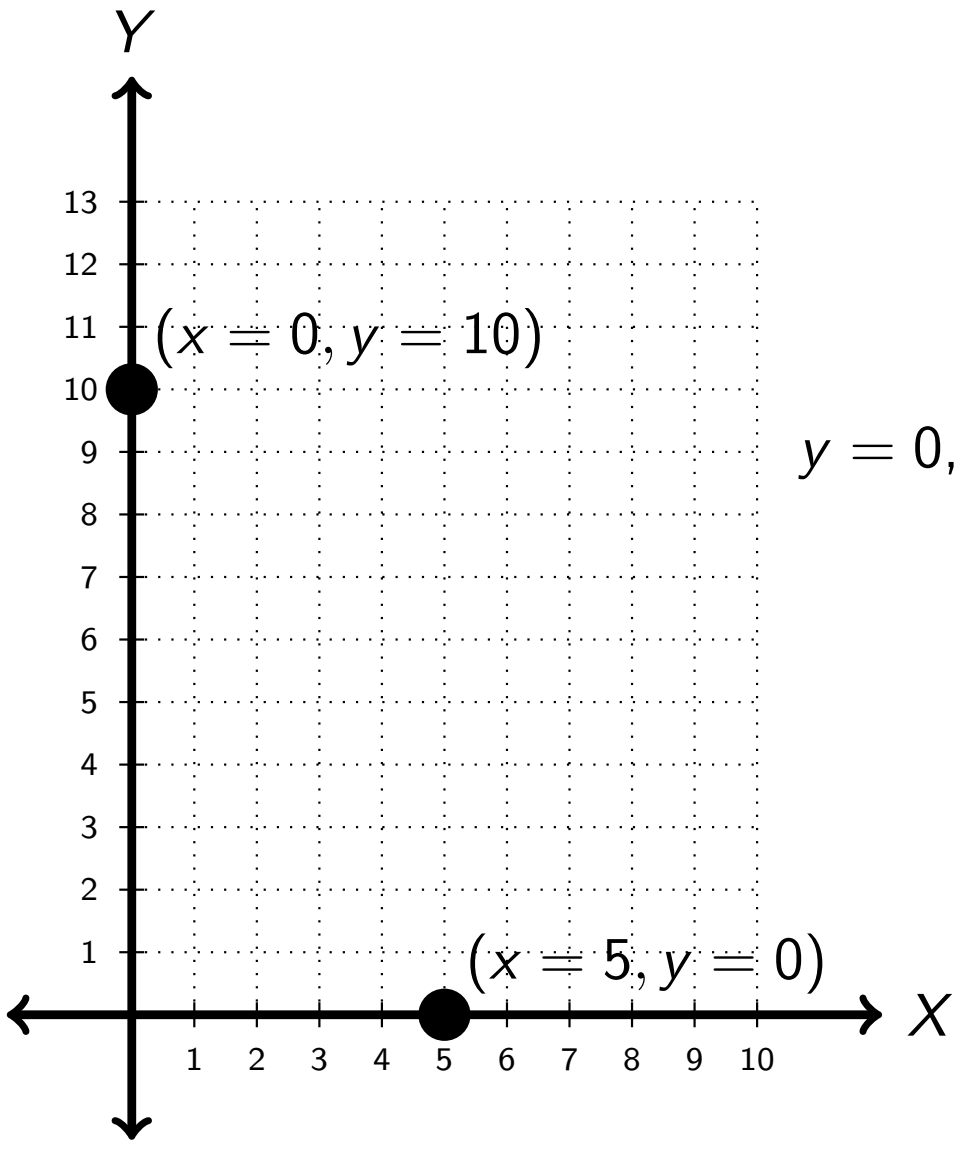


$200x + 100y = 1000$

$x = 0,\ 100y = 1000,\ y = 10$

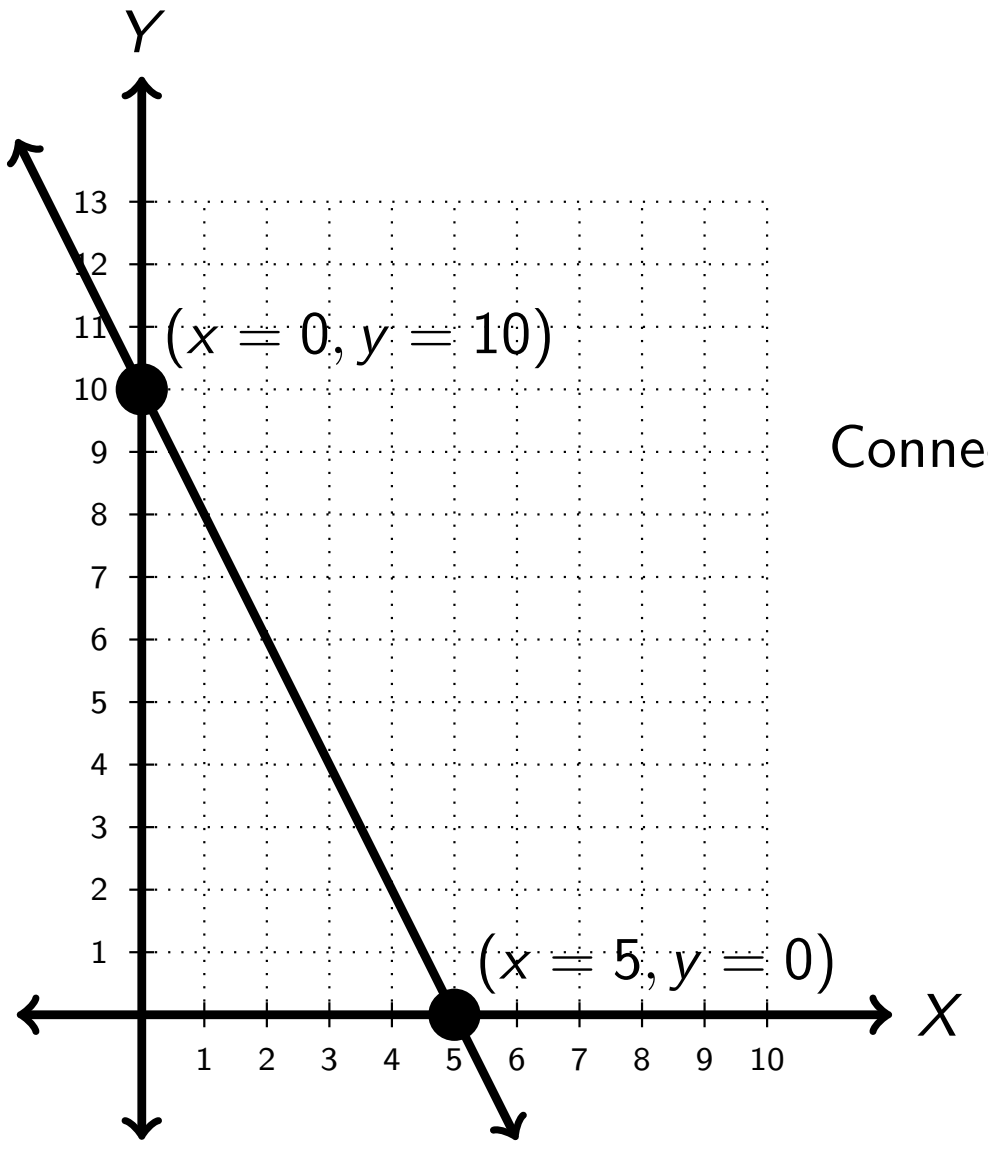
3.1: Graphing inequalities



$$200x + 100y = 1000$$

$$y = 0,\ 200x = 1000,\ x = 5$$

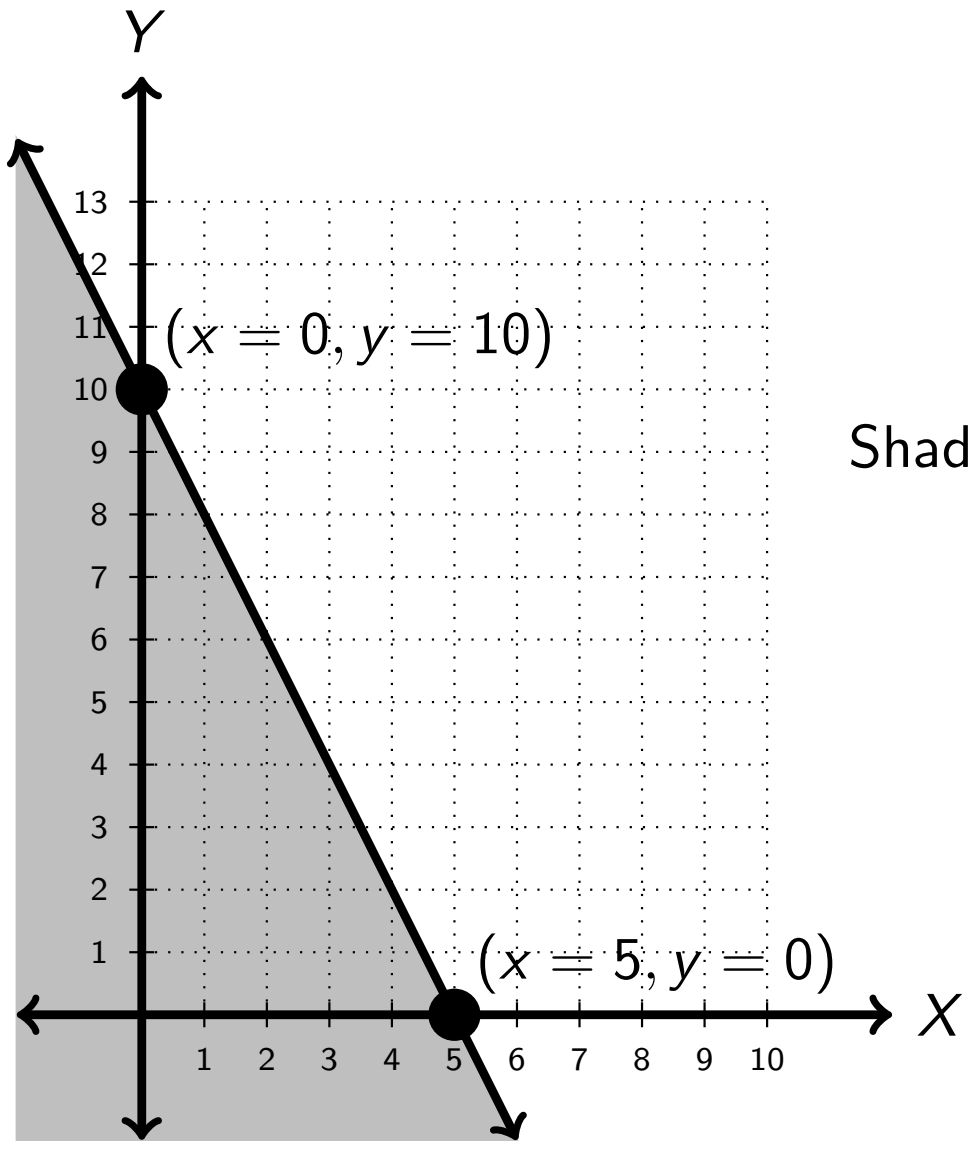
3.1: Graphing inequalities



$200x + 100y = 1000$

Connect the dots

3.1: Graphing inequalities



$200x + 100y \leq 1000$

Shade the region

3.1: Graphing inequalities

- First graph the "equality", that is, graph the line

$\Rightarrow$ Find two points on the line and then draw the connection

- Next graph the inequality, that is, shade the region

$\Rightarrow$ Choose a point not on the lines and see if it is on the correct side

- For example (0,0) is on the correct side since

$$(200)(0) + (100)(0) \leq 1000$$

3.1: Is it realistic?

- Our region is very large.
- Some points don't make sense for a single purchaser:

$\Rightarrow$ $(2.5, 3.5)$ means buy 2.5 Xylophones and 3.5 Yukuleles ($850)

- But maybe it makes sense as an average or a strategy
- Some points don't make any sense for any purchaser:

$\Rightarrow$ $(-10, -20)$ means buy -10 Xylophones . . . (-$4000)

3.1: Systems of inequalities

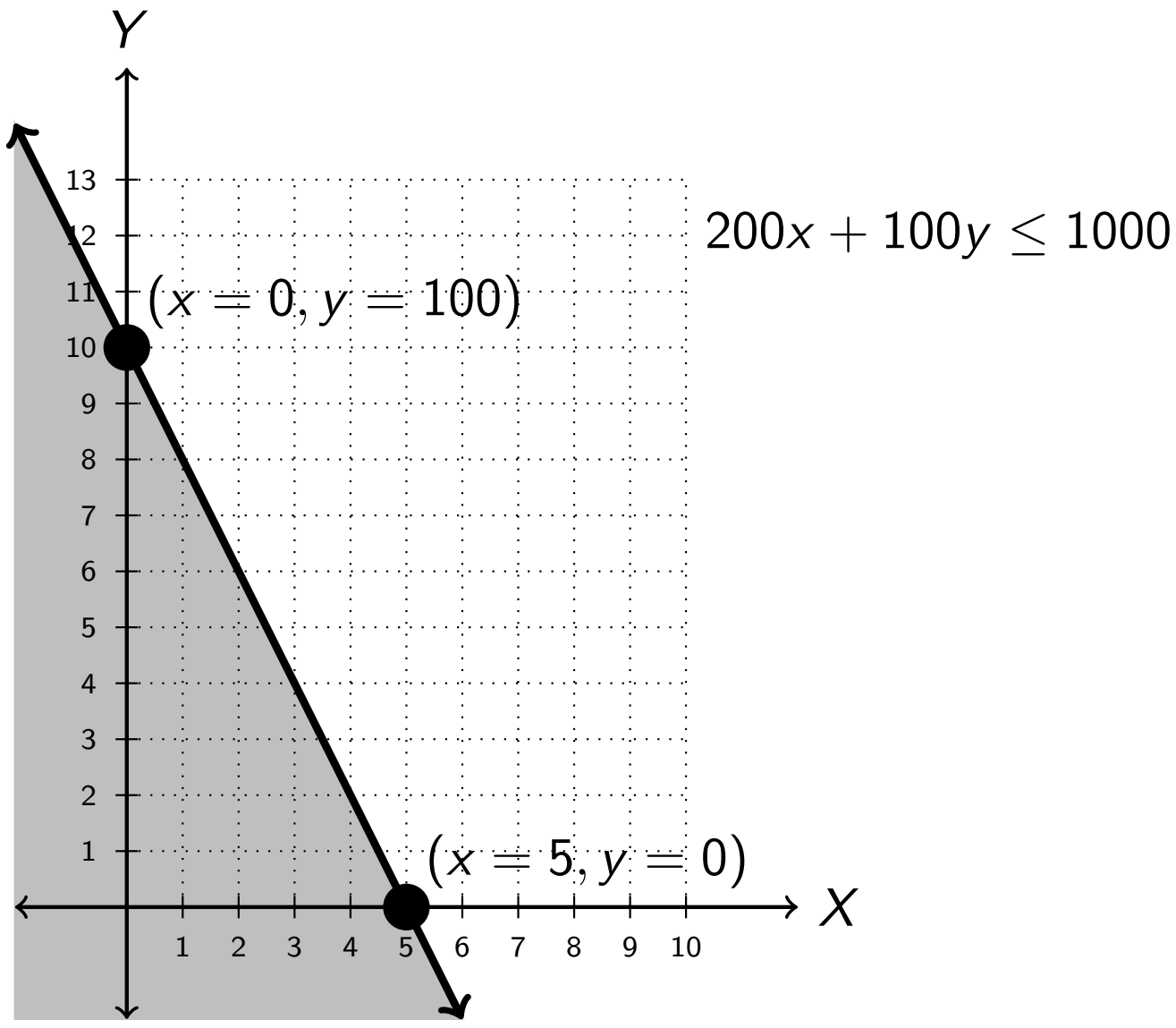
- We also need some sanity: $X \geq 0$ and $Y \geq 0$
- So we have a system of inequalities:

$$\begin{cases} 200X + 100Y \leq 1000 \\ X \geq 0, Y \geq 0 \end{cases}$$

- Not enough for just one to be true!

$\Rightarrow$ $(500, 0)$ would be very expensive ($100,000) and noisy!

3.1: Graphing systems of inequalities



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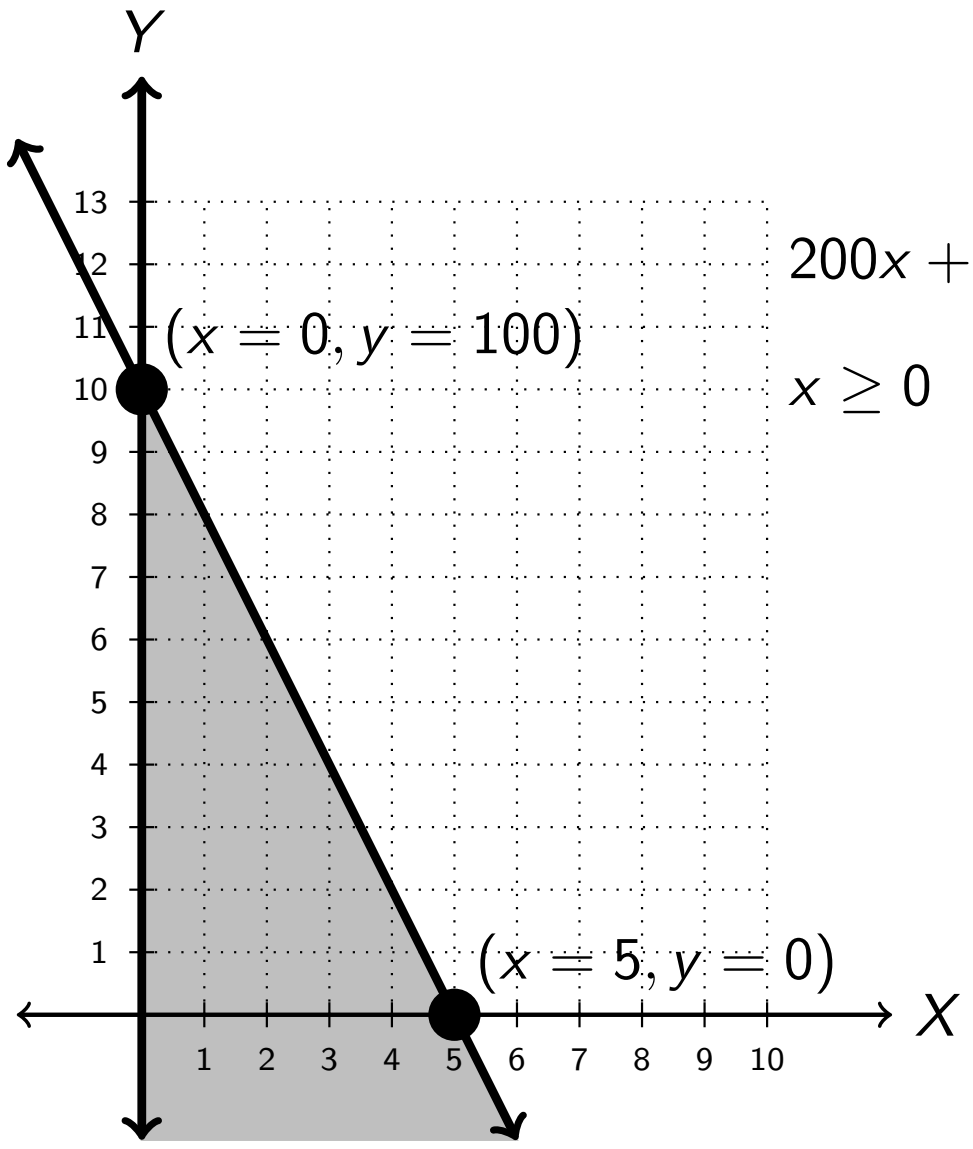

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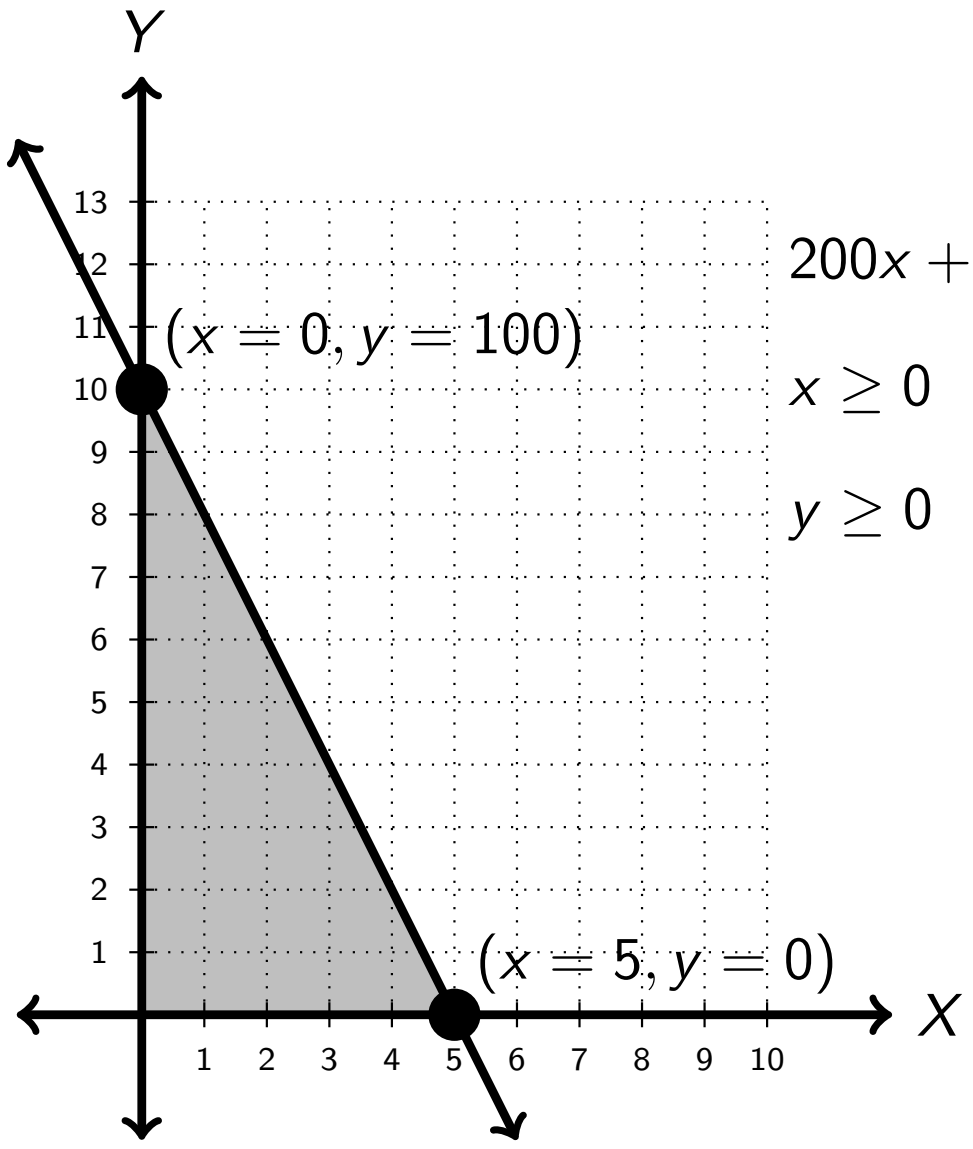


$200x + 100y \leq 1000$

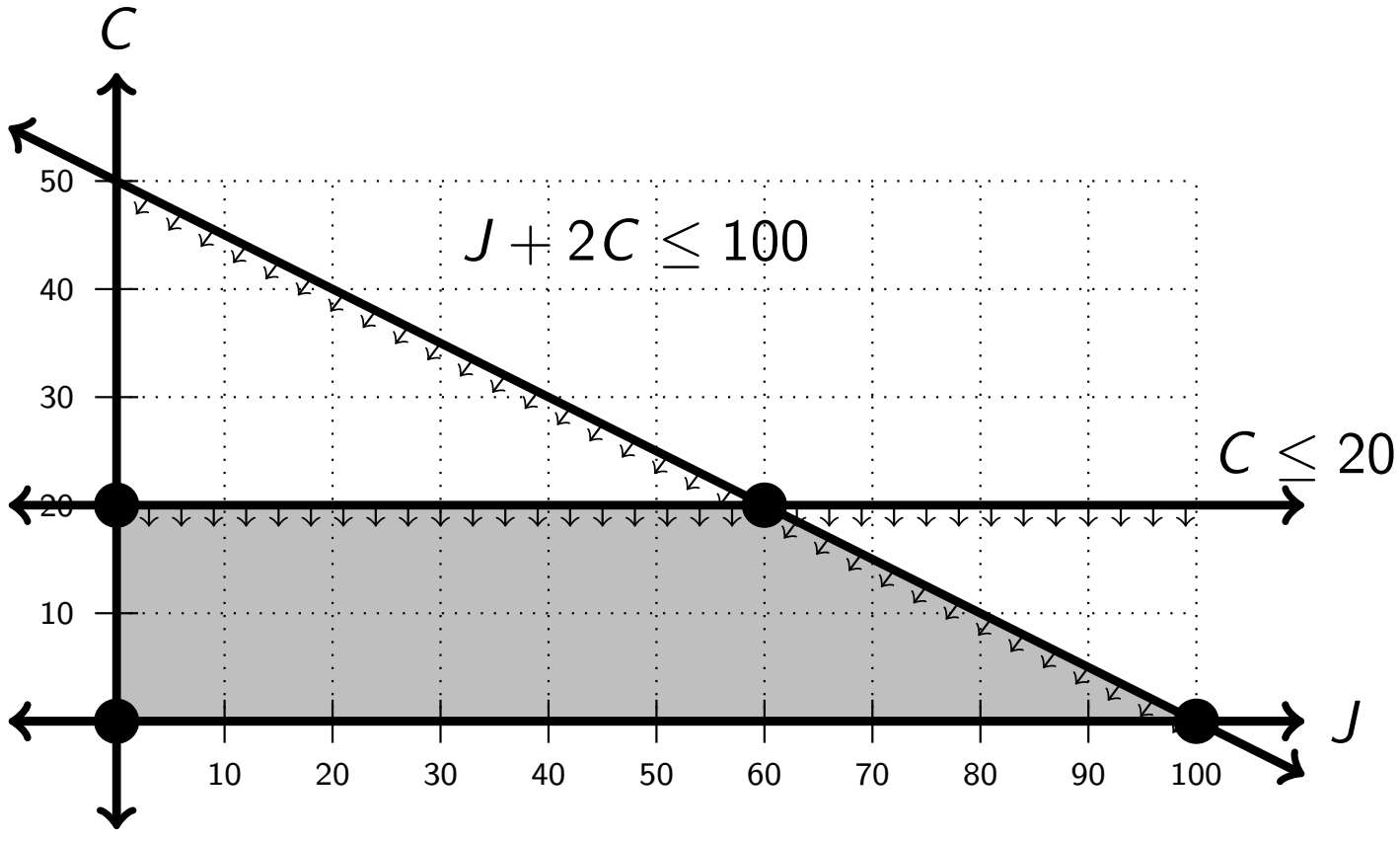
$x \geq 0$

$y \geq 0$

3.1: Graphing systems of inequalities

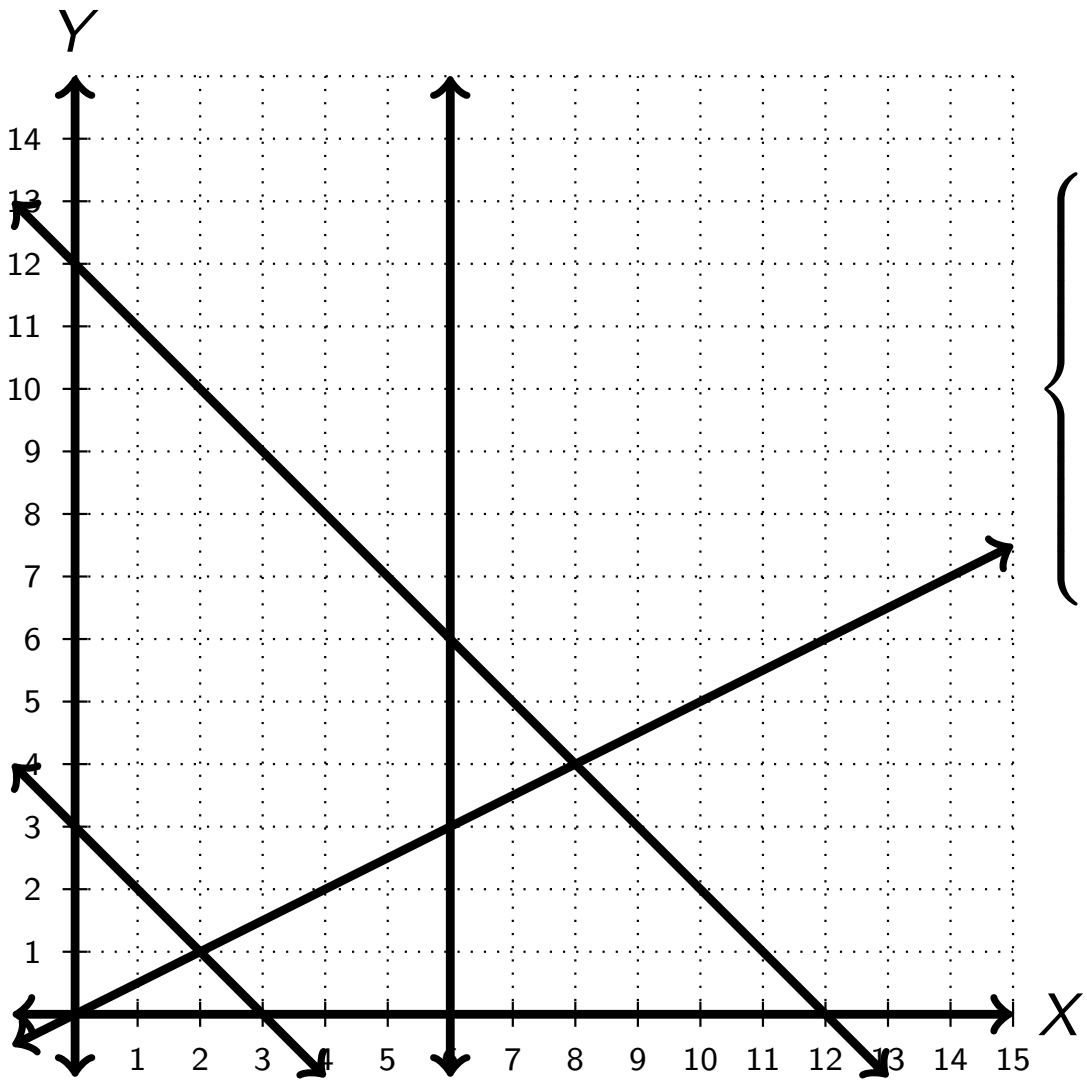
- Graph each equality (line)
- Figure out which side of the line is good
- Shade the region that is on the correct side of **all** lines
- Alternatively: figure out which of the pieces is good

3.1: Graphing systems of inequalities



Draw little arrows to show which side is good.

3.1: Graphing systems of inequalities



$$\begin{cases} x + y = 12 \\ x - 2y = 0 \\ x + y = 3 \\ x = 6 \\ x = 0 \\ y = 0 \end{cases}$$

Draw all the lines, then check each inequality.

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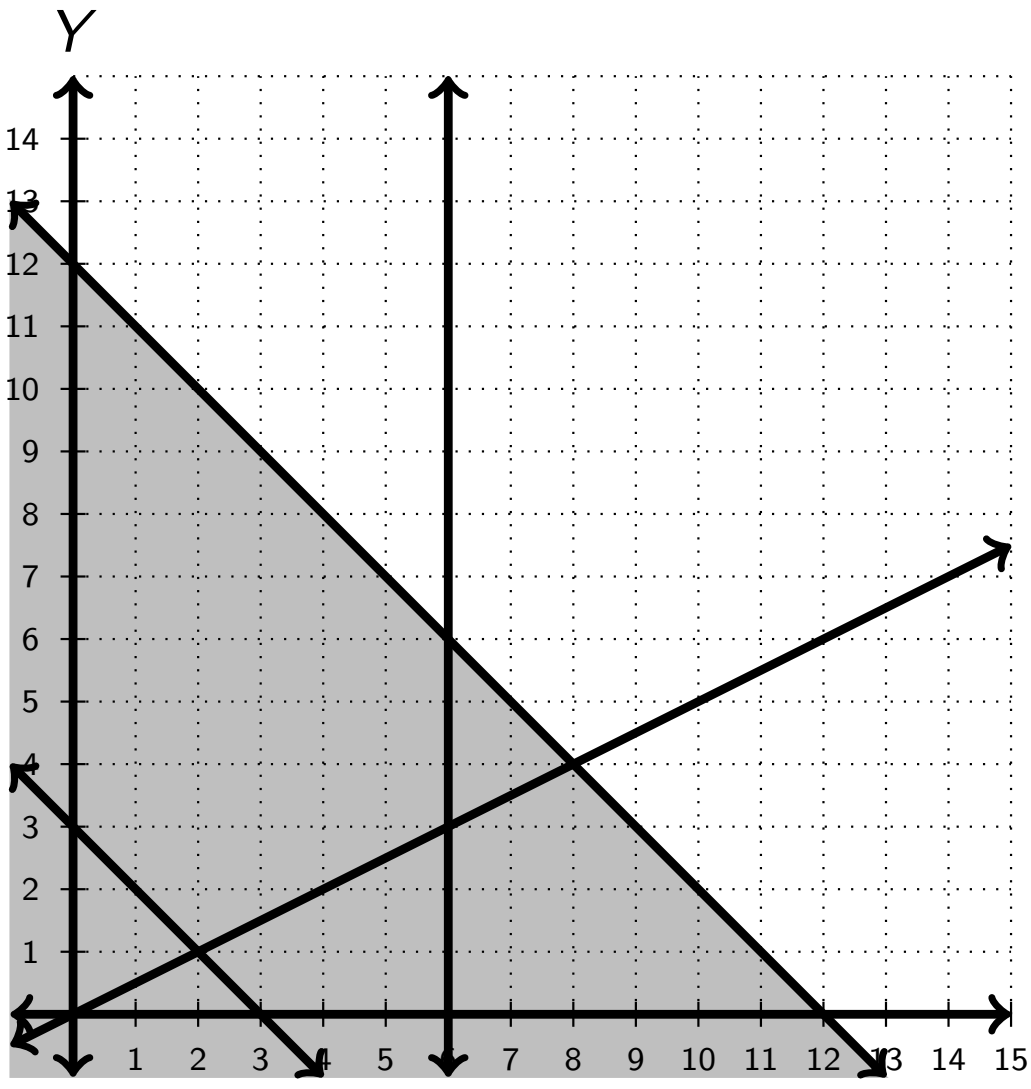

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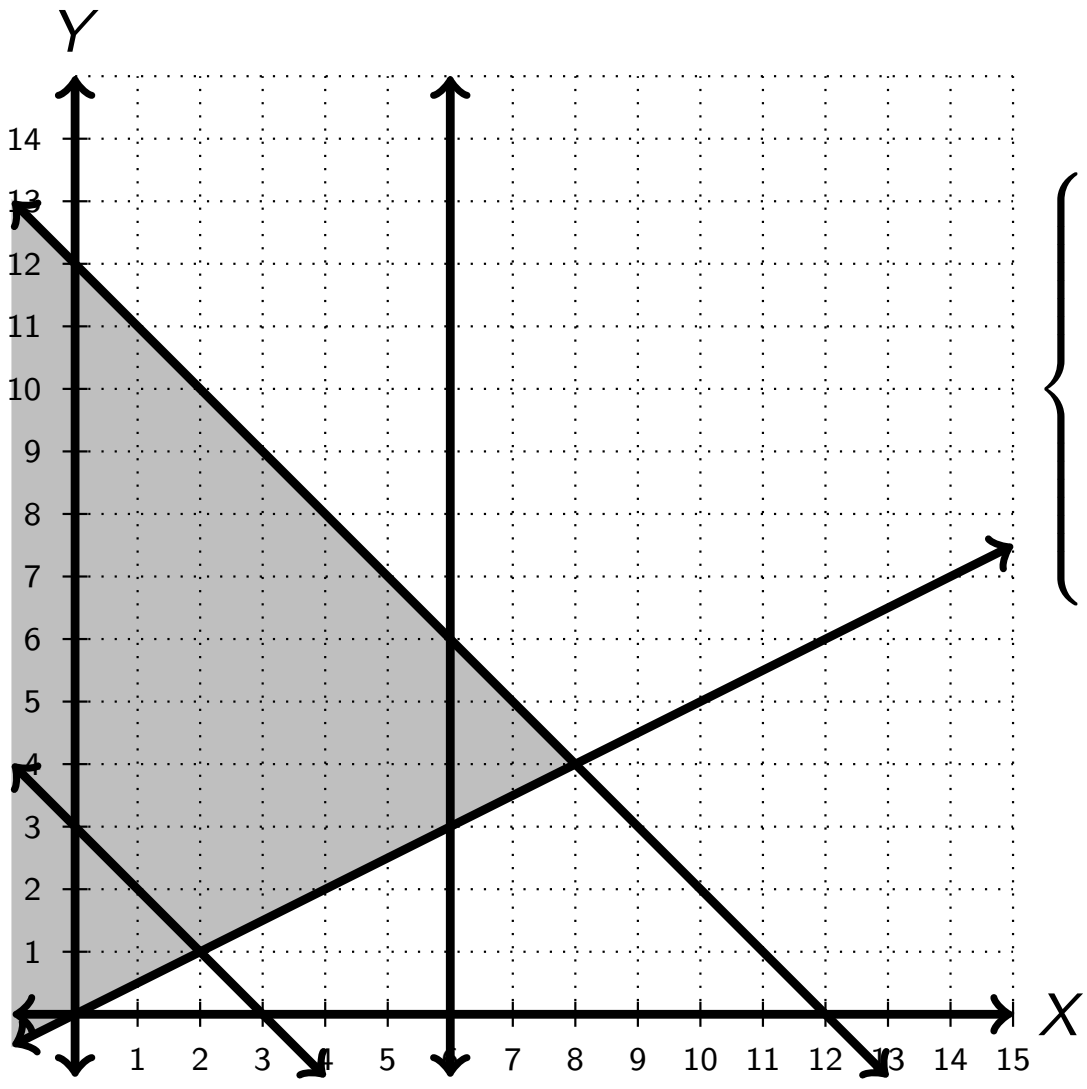

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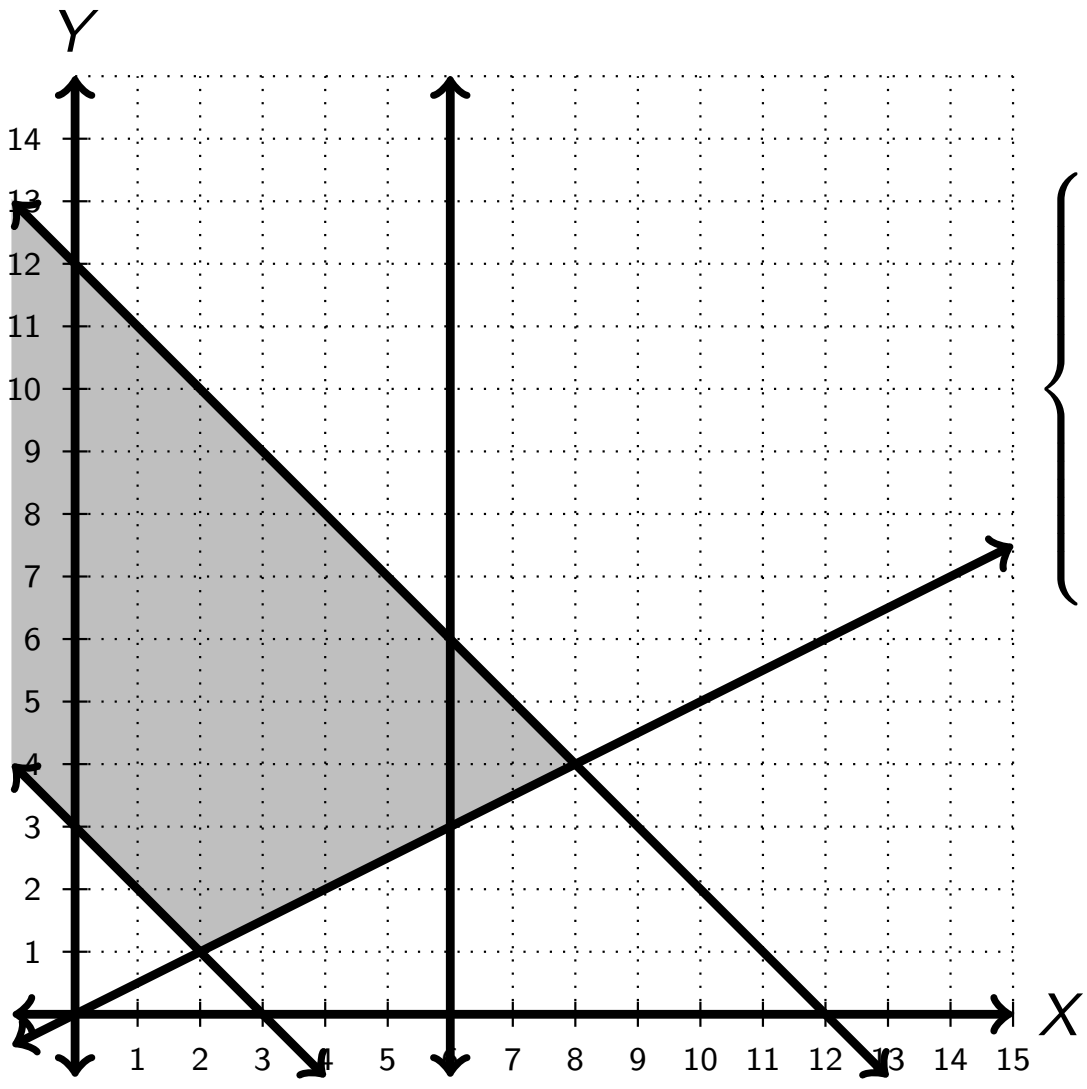

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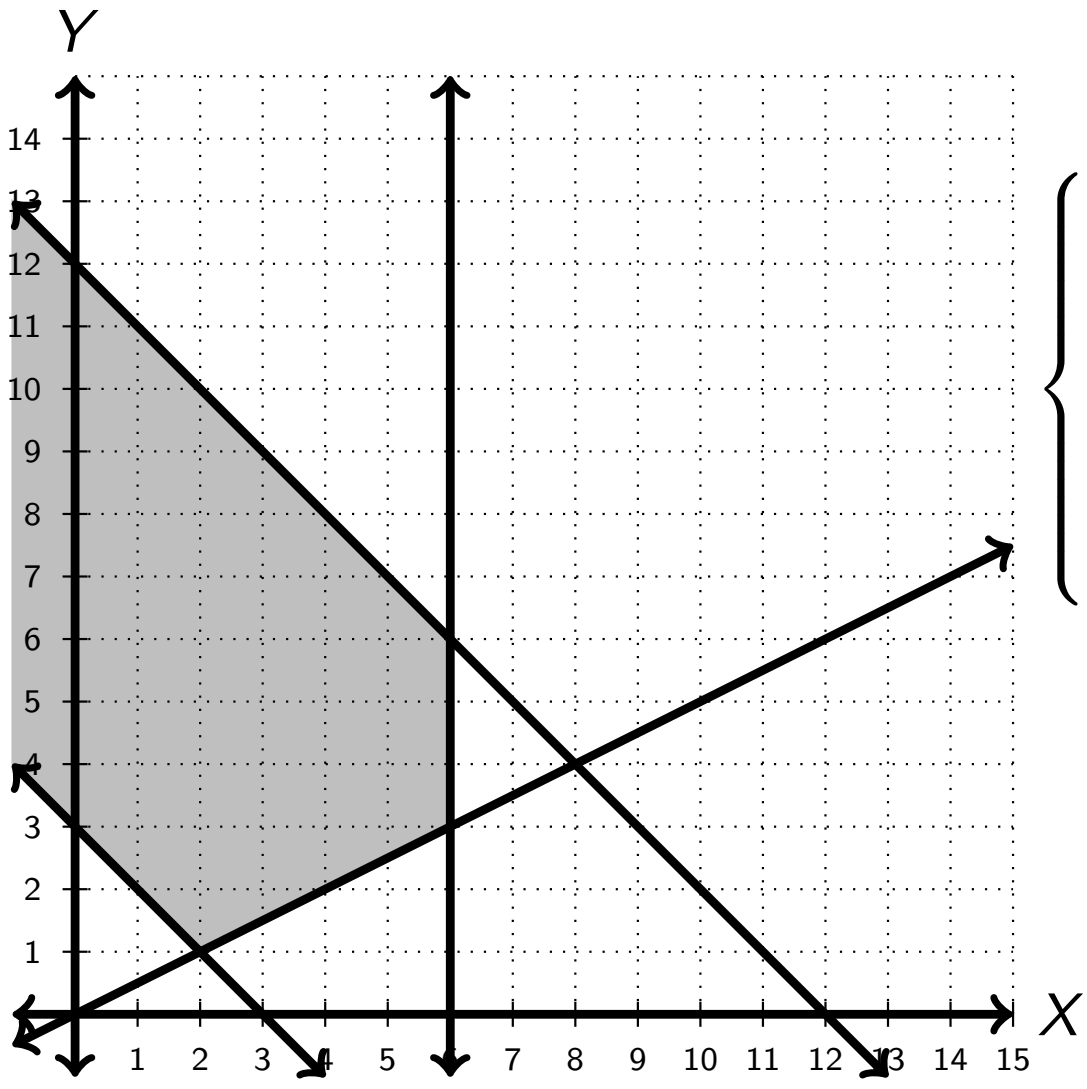

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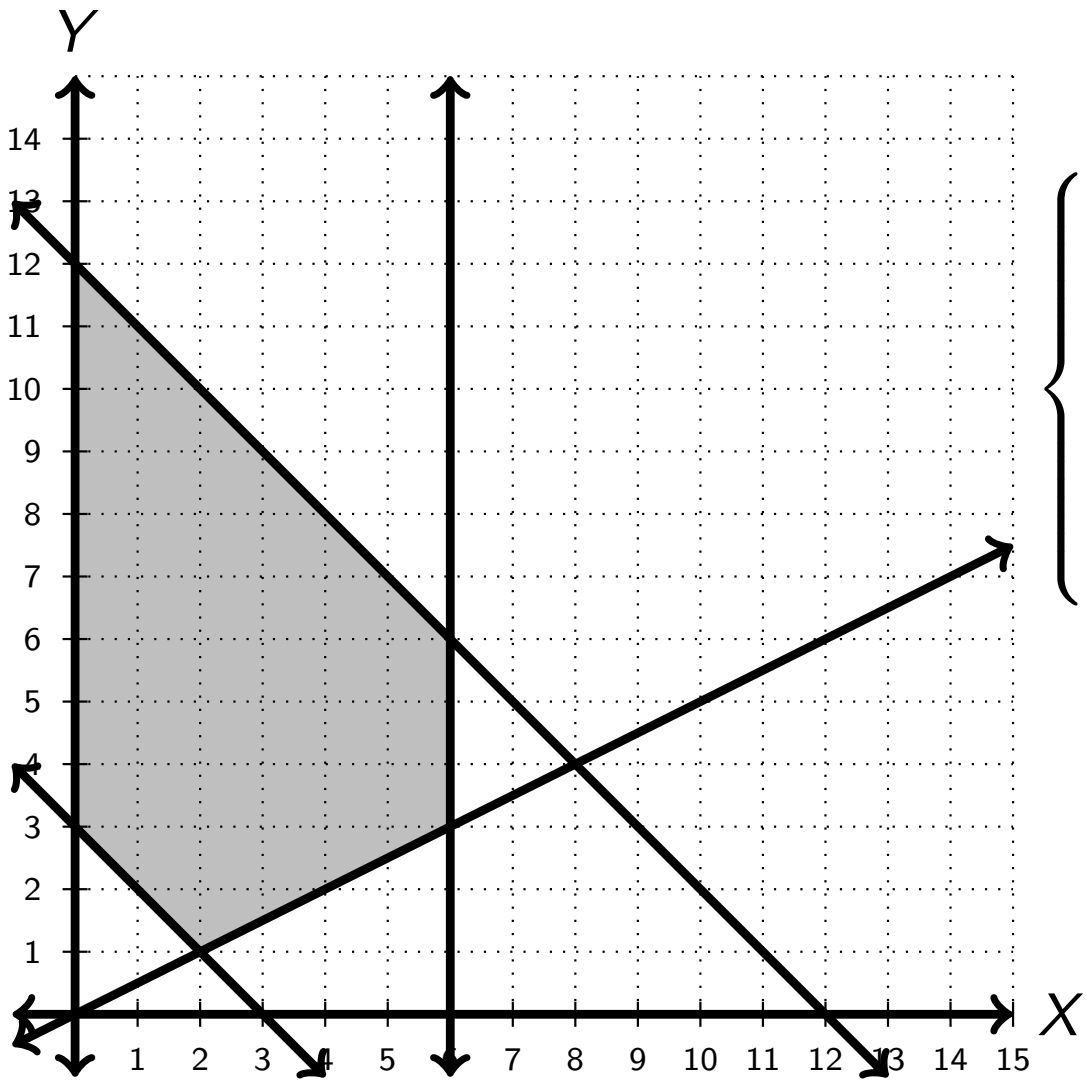

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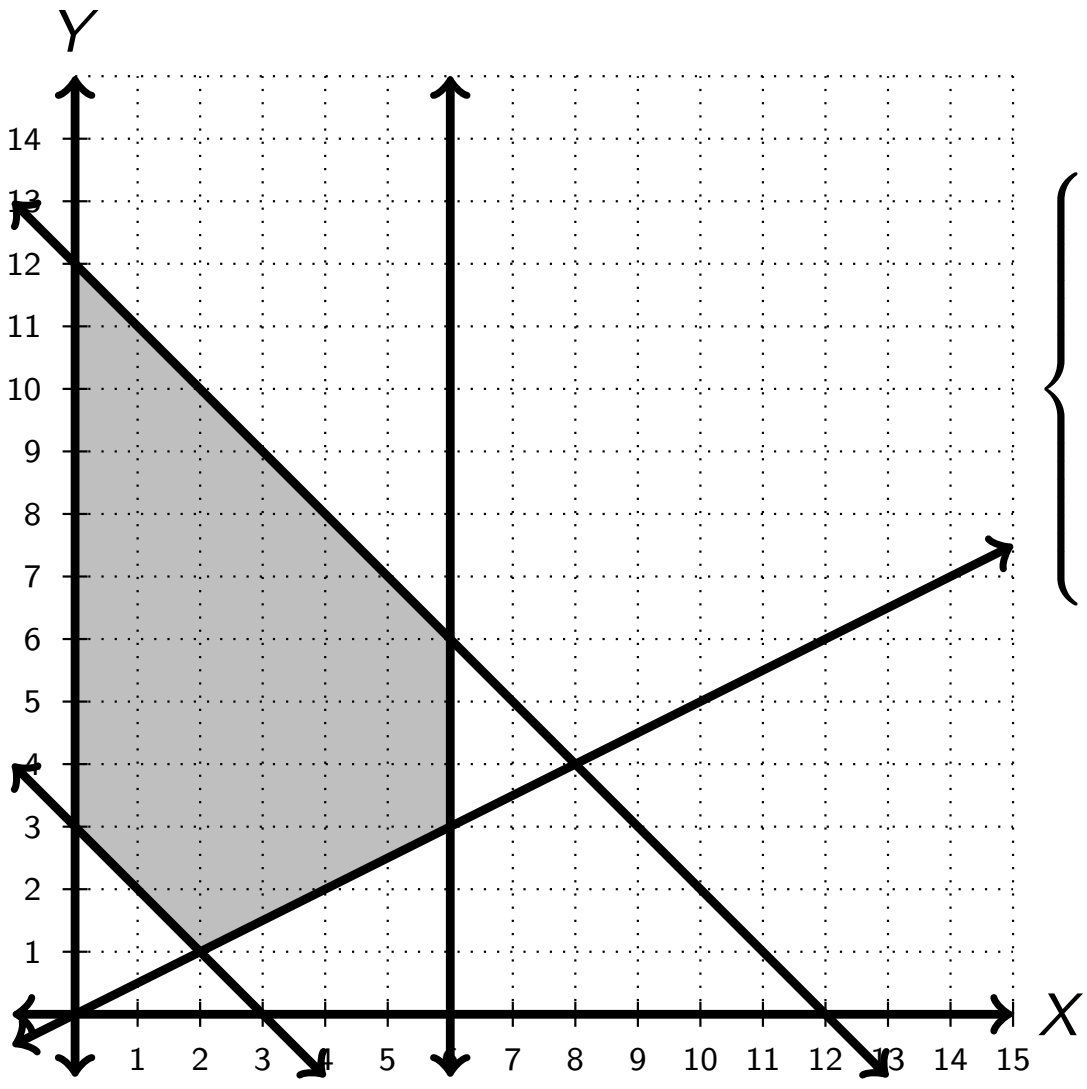

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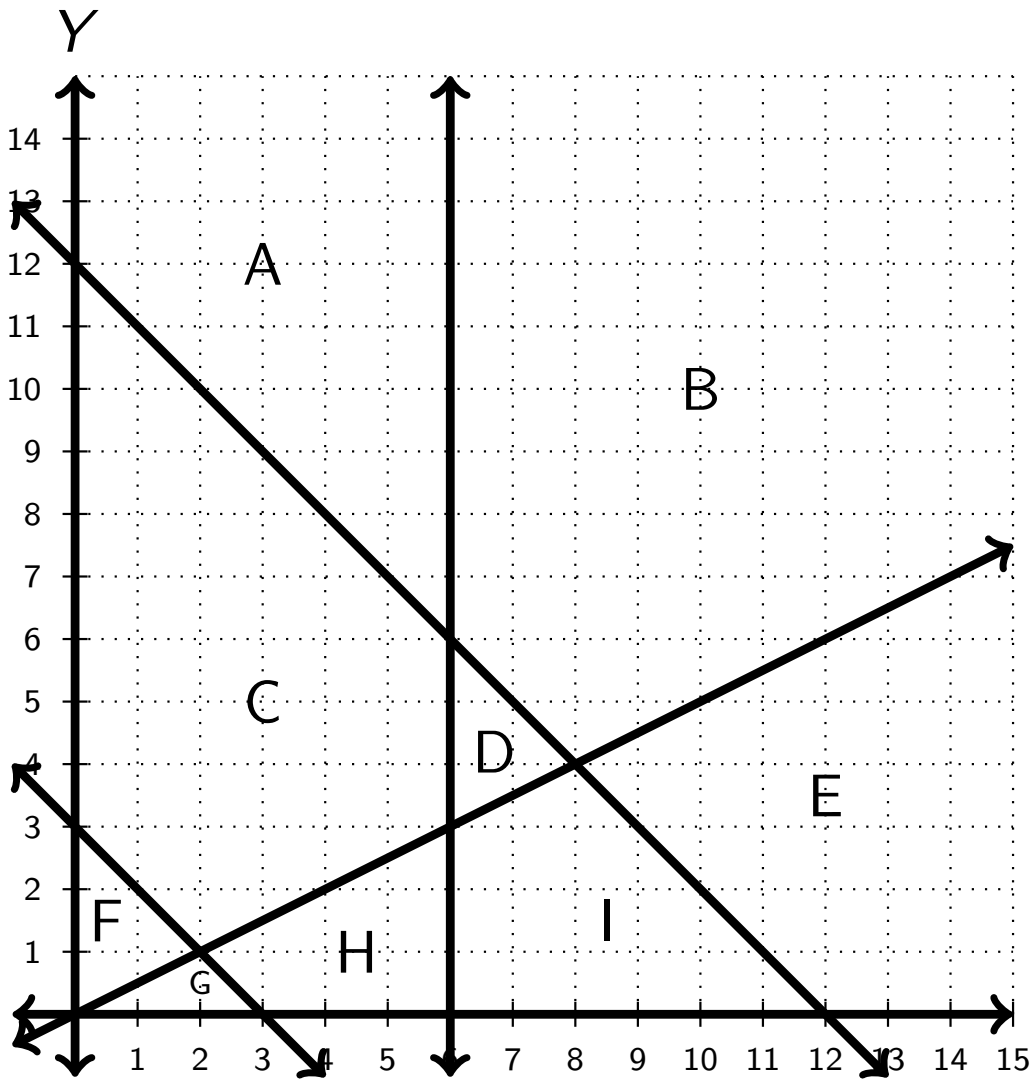
Draw all the lines, then check each inequality. Too many regions!

3.1: Graphing systems of inequalities



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Check a point in each region to find the right one.

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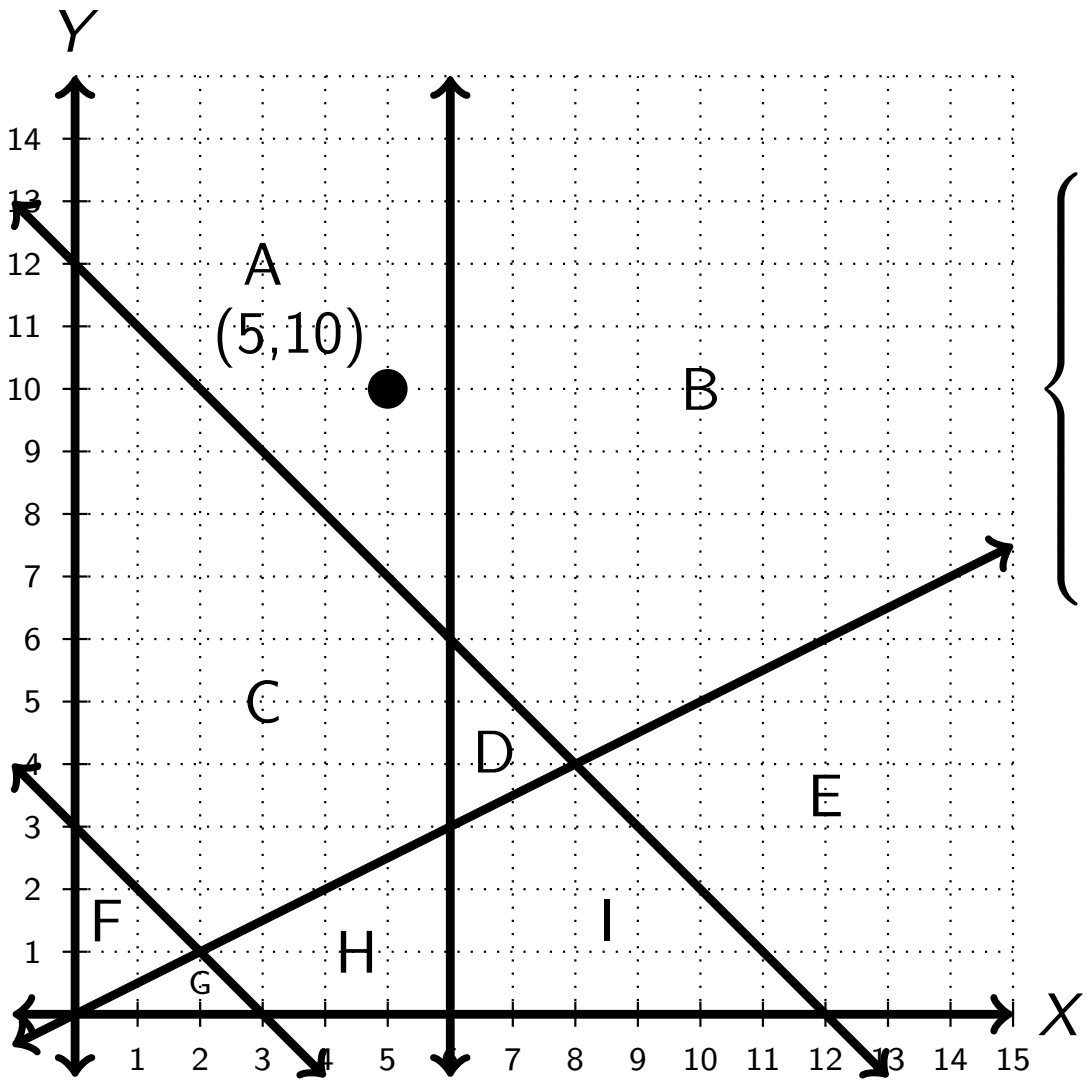

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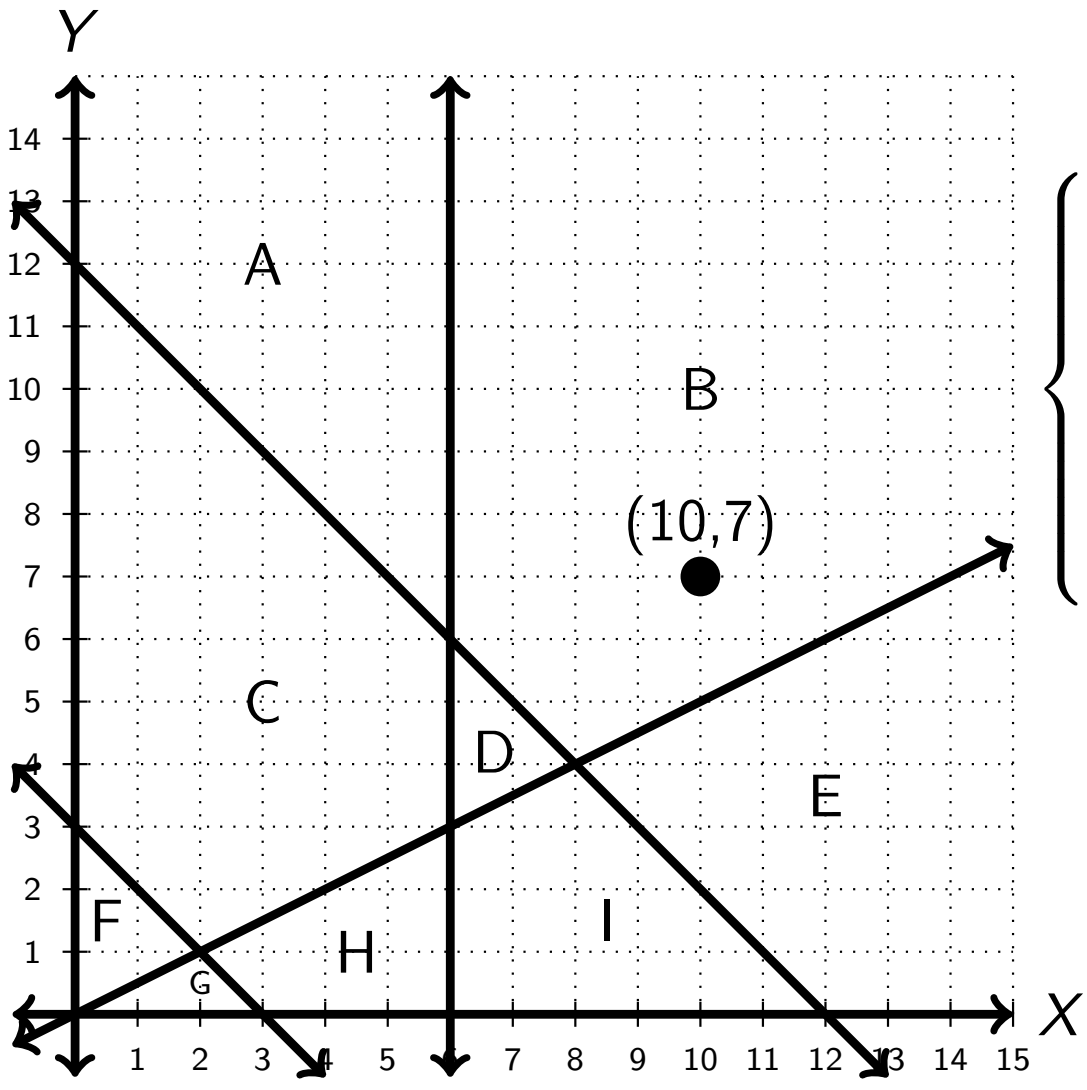

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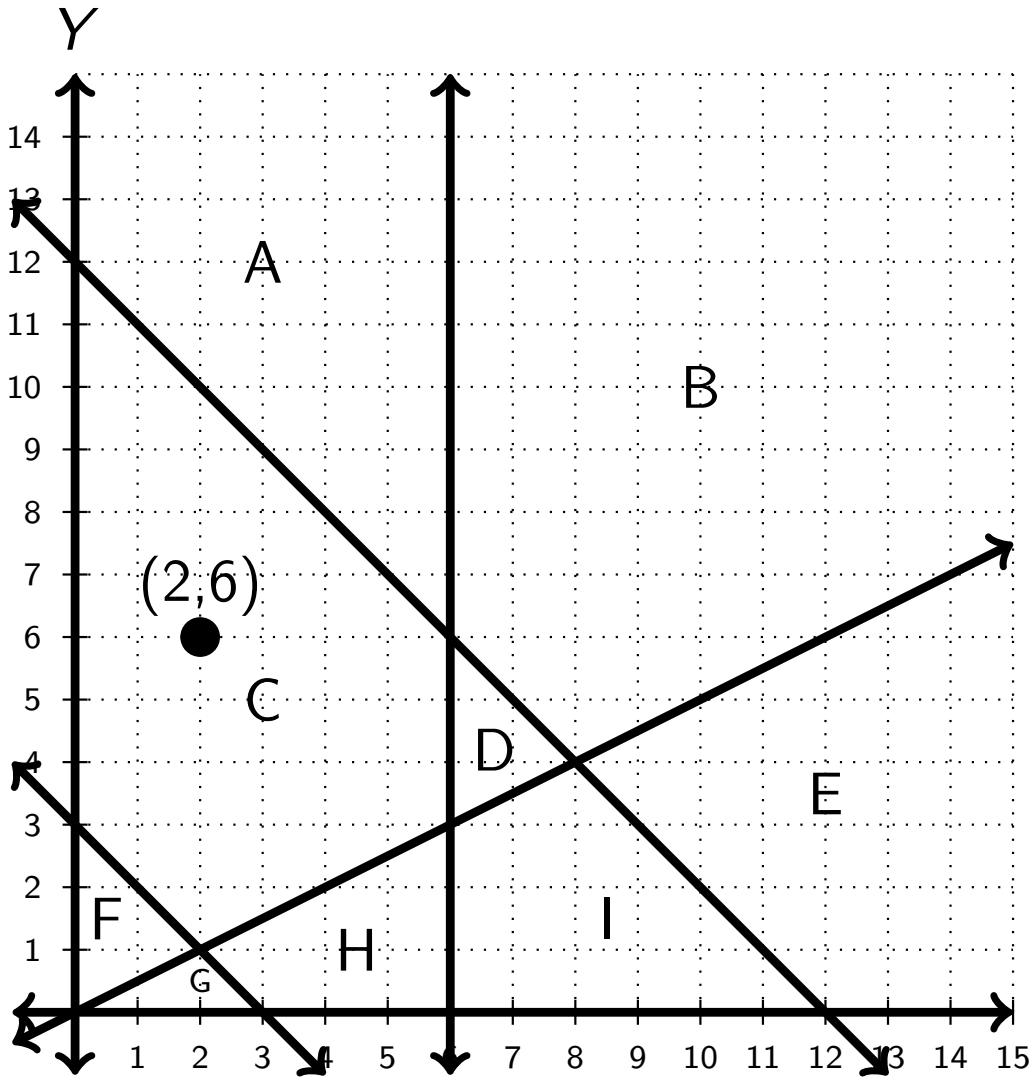

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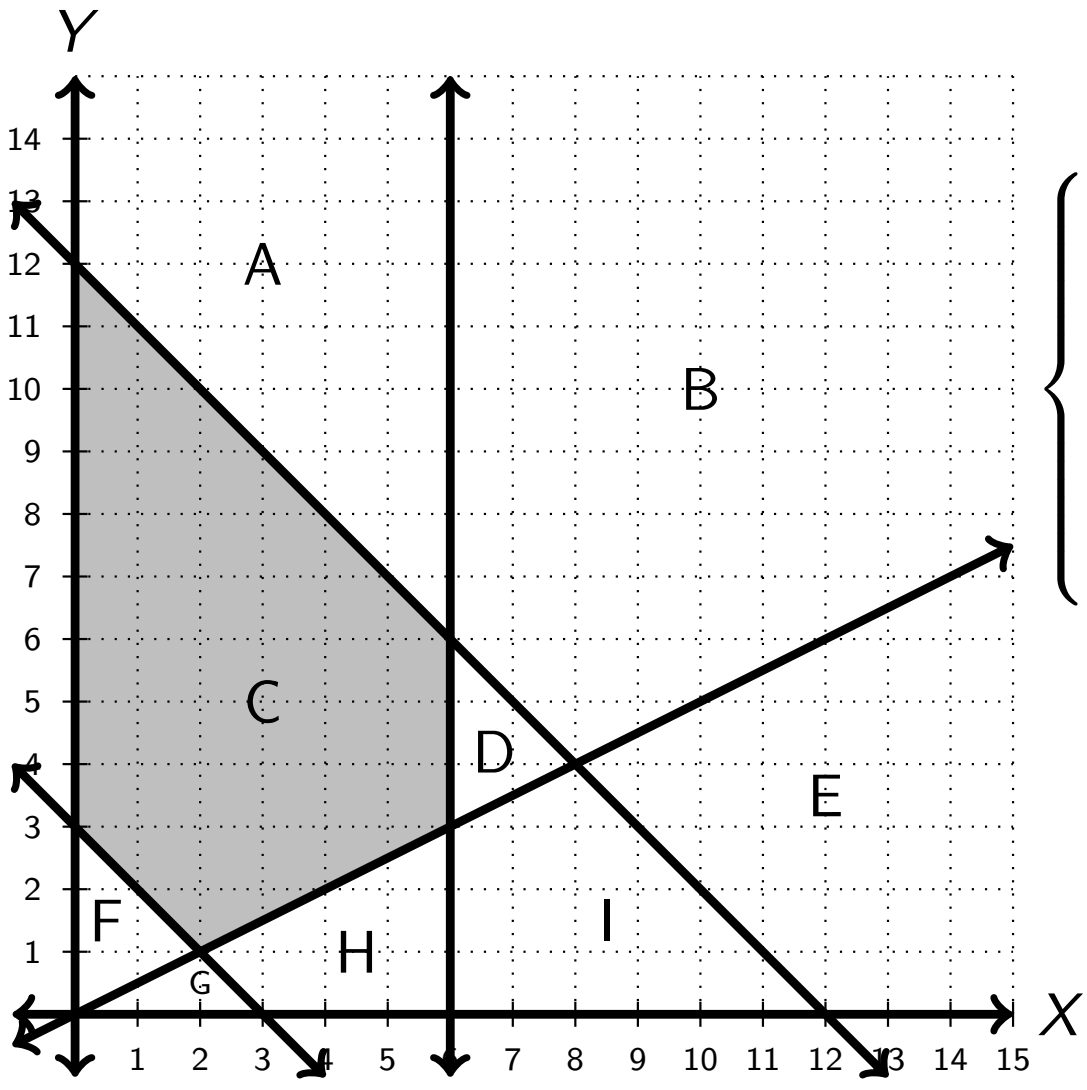
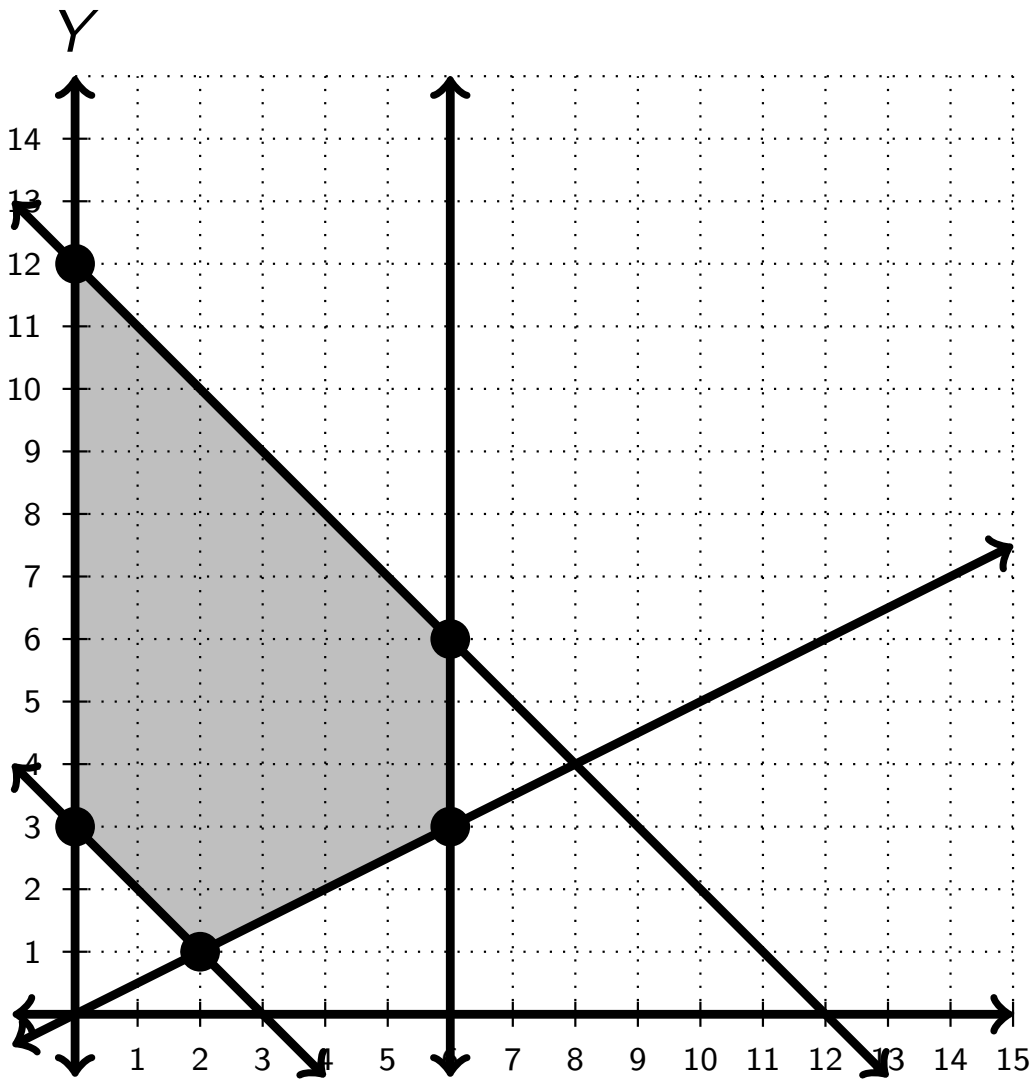


$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Check a point in each region to find the right one. Yay!

3.1: Finding corners



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Intersect each pair of lines, and check it satisfies other inequalities

3.1: Finding corners

- For each pair of lines, find the intersection
- Then check that intersection satisfies the rest of the inequalities
- Not all intersections are corners!
- All corners are intersections.
- Intersections are just 2×3 RREF problems!

3.1: How many corners are there?

- How many angles does a triangle have?

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- How many angles does a triangle have?
- How many sides does a triangle have?

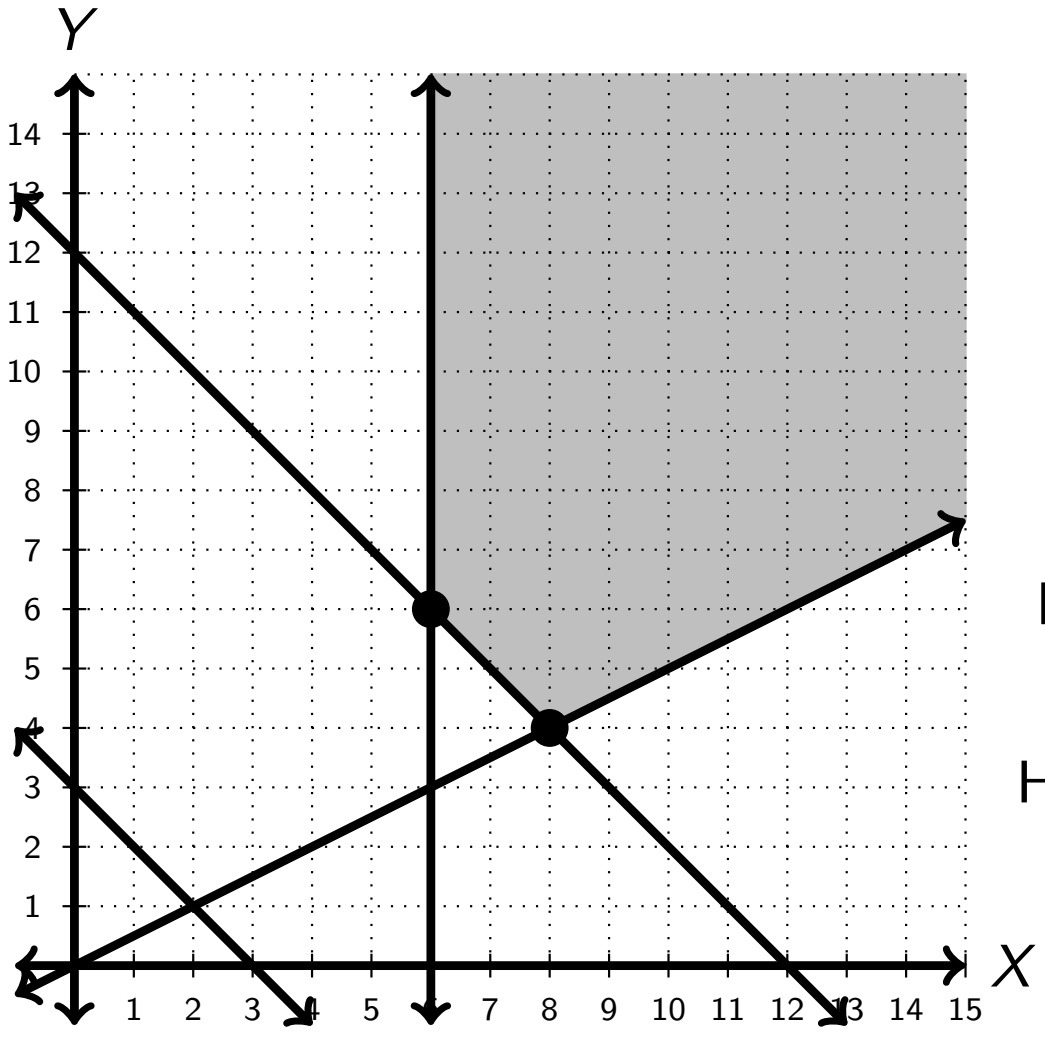
3.1: How many corners are there?

- How many angles does a triangle have?
- How many sides does a triangle have?
- How many angles does a quadrangle have? A quadrilateral?

3.1: How many corners are there?

- How many angles does a triangle have?
- How many sides does a triangle have?
- How many angles does a quadrangle have? A quadrilateral?
- An n-sided polygon has n angles too!

3.1: Where's the missing corner?



How many edges?

How many corners?

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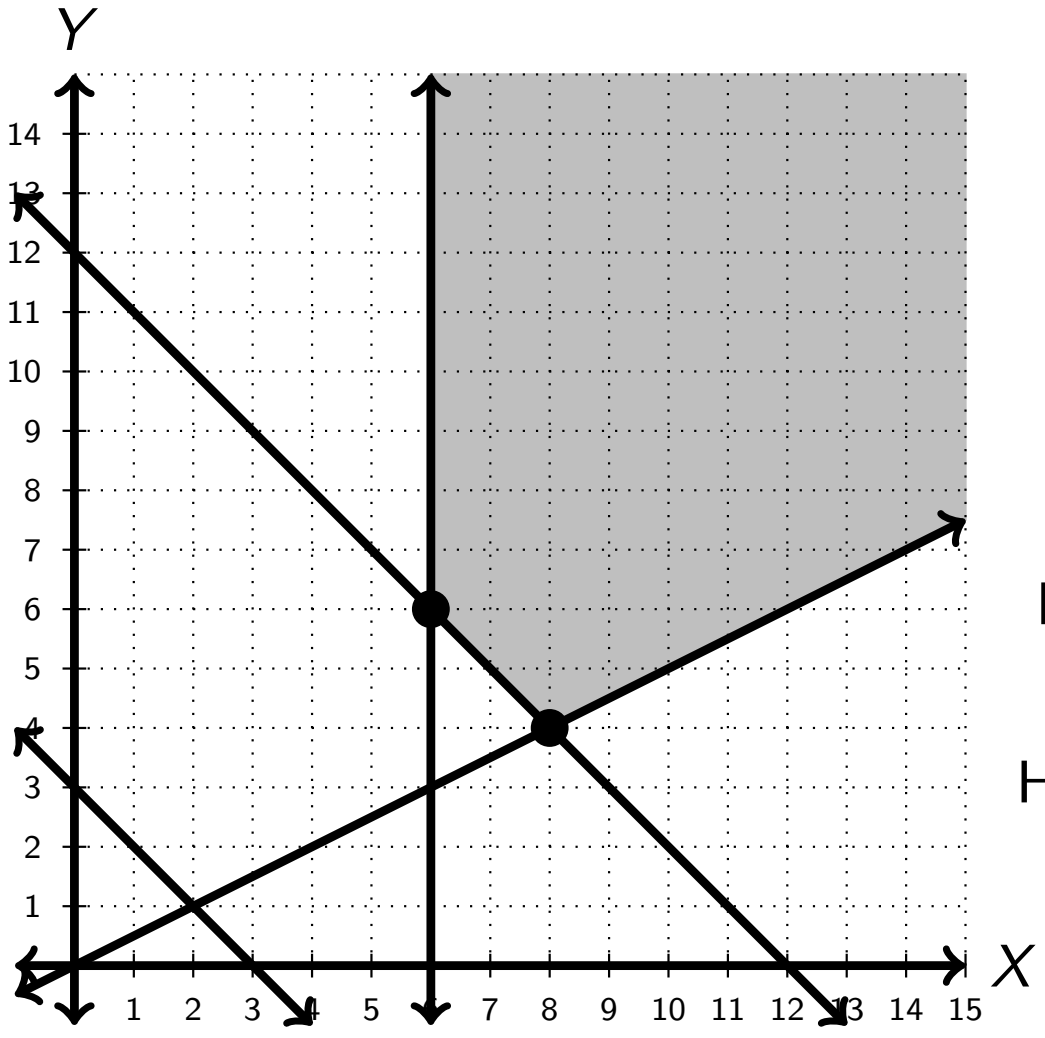

How many edges?

How many corners?

- This is called **unbounded** and it means we need to handle the "missing corner" specially.