MA162: Finite mathematics

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University of Kentucky

November 5, 2012

Schedule:

- HW 6B,6C are due Fri, November 9th, 2012
- Exam 3 is Monday, November 12th, 5pm to 7pm in BS107 and BS116
- Exam 2 grades on Blackboard, PDFs on mathclass

Today we will cover 6.3: Multiplication principle

Exam 3 breakdown

- Chapter 5, Interest and the Time Value of Money
  - Simple interest
  - Compound interest
  - Sinking funds
  - Amortized loans

- Chapter 6, Counting
  - Inclusion exclusion
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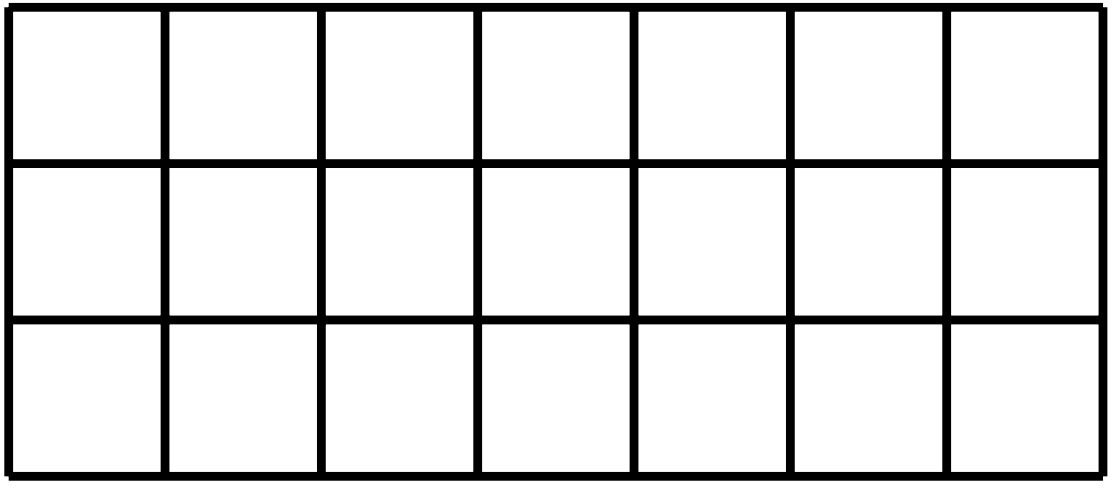
  - **Multiplication principle**
  - Permutations and combinations

6.3: What is multiplication?

- How many squares in this figure?



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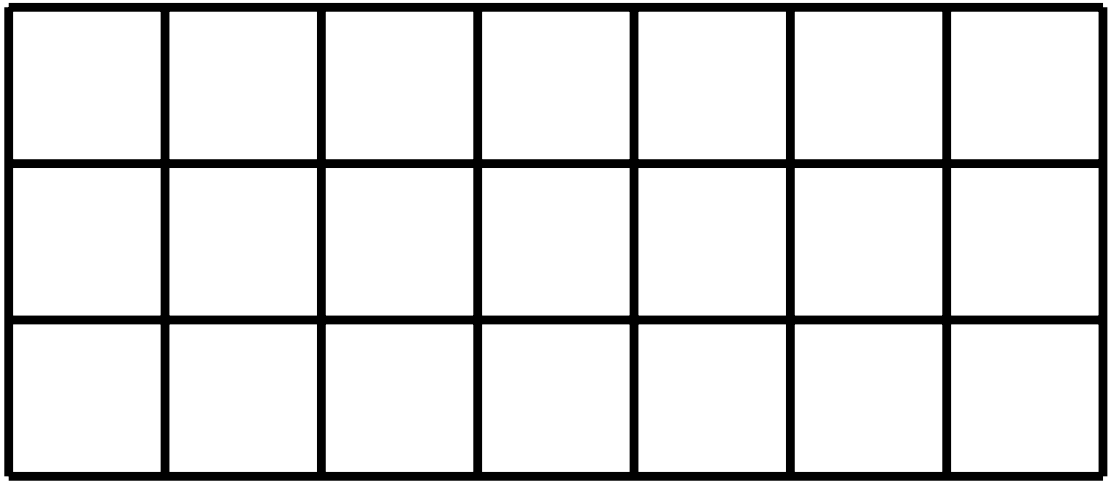

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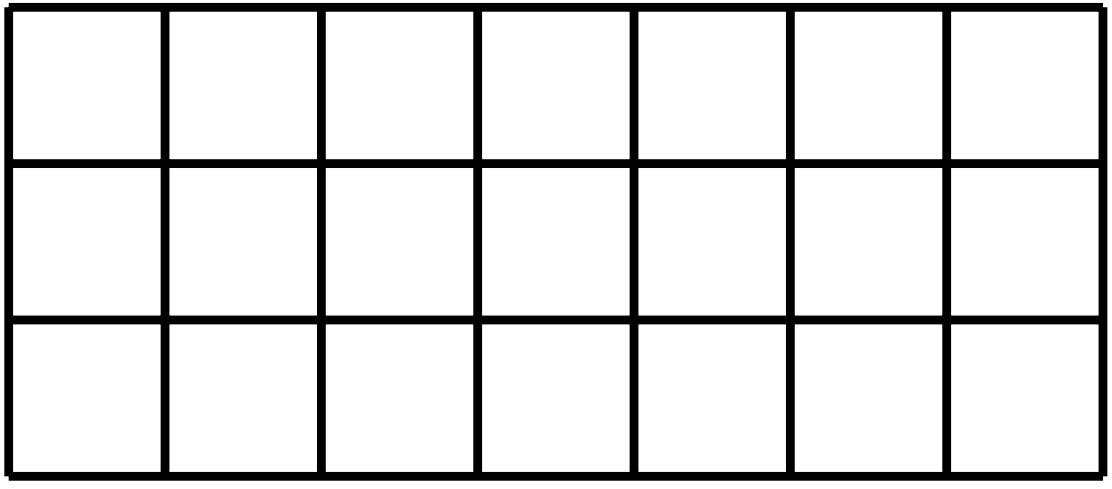

- Each column has 3 squares, there are 7 columns, so $3 \cdot 7 = 21$
- Counting each square is slower and error-prone.

6.3: Three square meals a day

- You decide to brush your teeth after every meal, but are worried about the toothpaste consumption. You use about 1% of the tube every time you brush. How many weeks will it last?

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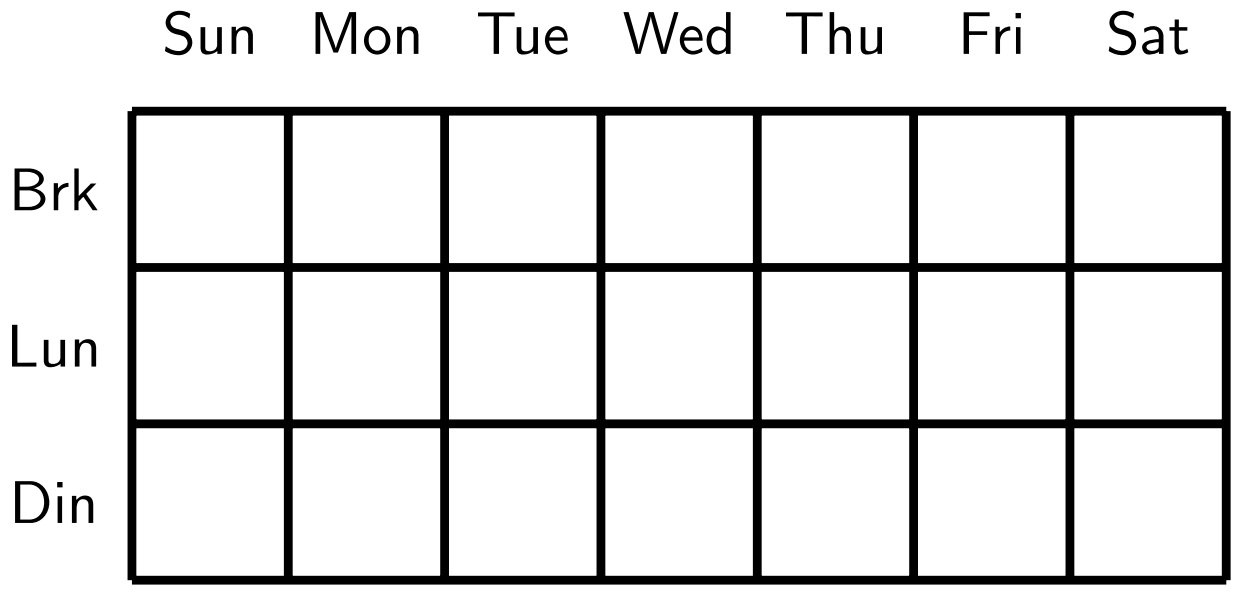

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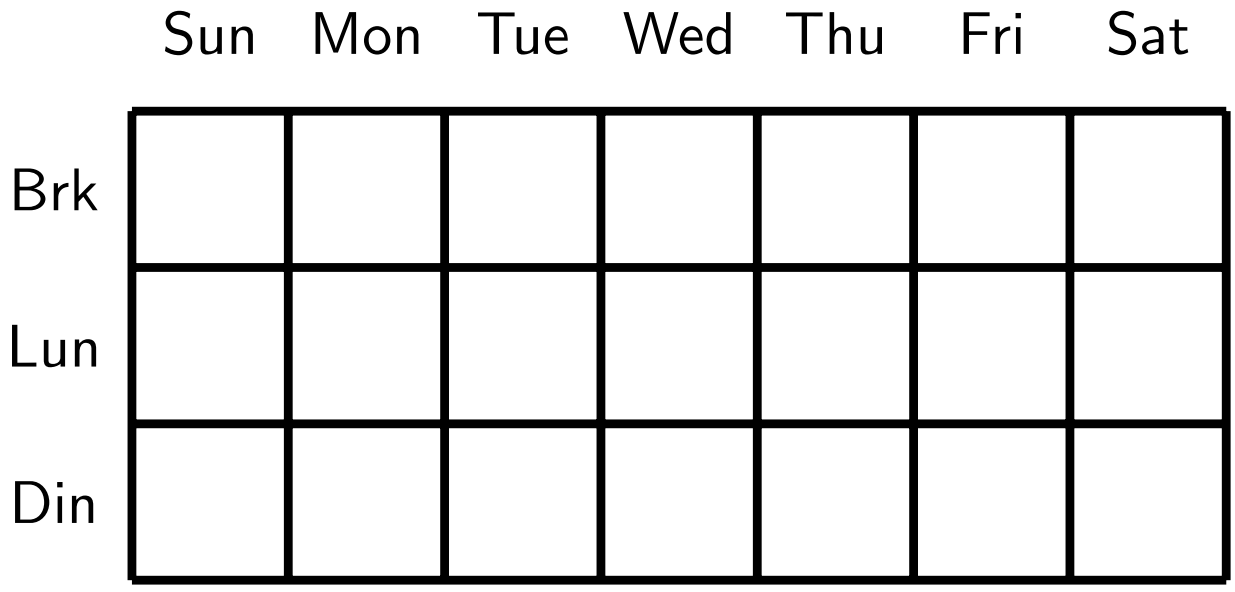

- So 21 brushes per week; takes less than 5 weeks to use up a tube.

6.3: A rainbow of possibilities

- You are working on a dazzling fashion project and have seven dyes: **Red**, **Orange**, **Yellow**, **Green**, **Blue**, **Indigo**, and **Violet**. You've got three types of fabric: Burlap, Cotton, and Denim.

  How many different color/texture combinations do you have?

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- Again $(3)(7) = 21$

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	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

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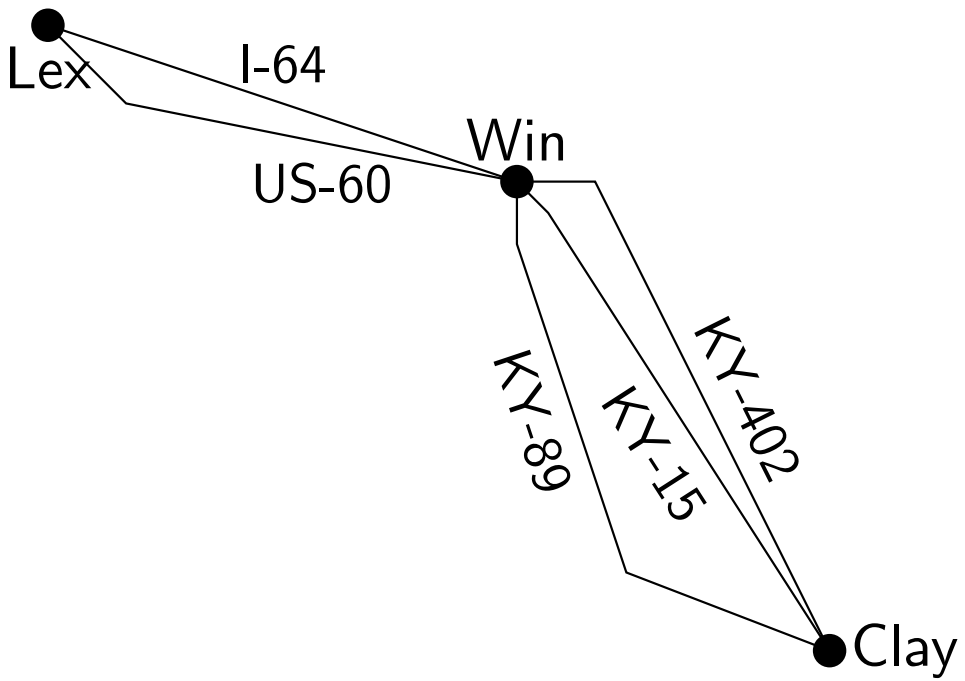
- **HHH**, **HHT**, **HTH**, **HTT**, **THH**, **THT**, **TTH**, **TTT**
  (2)(2)(2)=8

6.3: Drawing the possibilities

- There are two main ways to get to Winchester from Lexington: Winchester Rd (US-60) and I-64.
  From Winchester, there are three main ways to Clay City: KY-89, KY-15, and the Mountain Parkway (KY-402).
  How many different ways are there from Lexington to Clay City using these routes?



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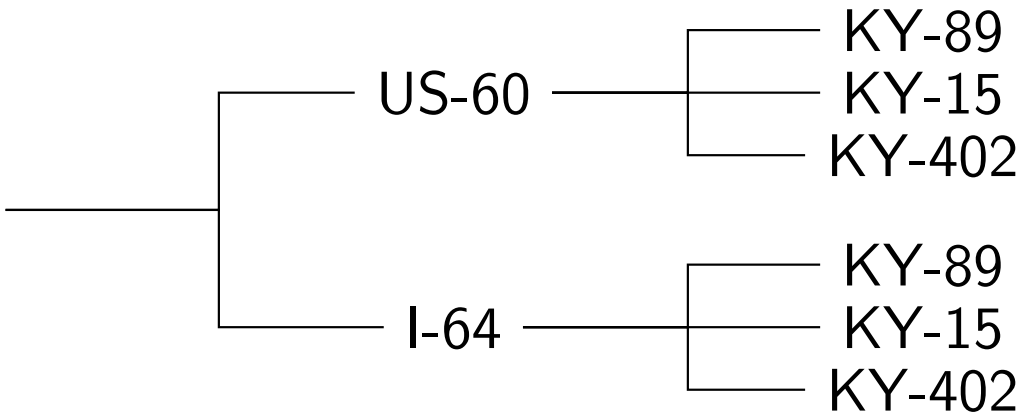
- We can unfold the map to make the possibilities clearer:



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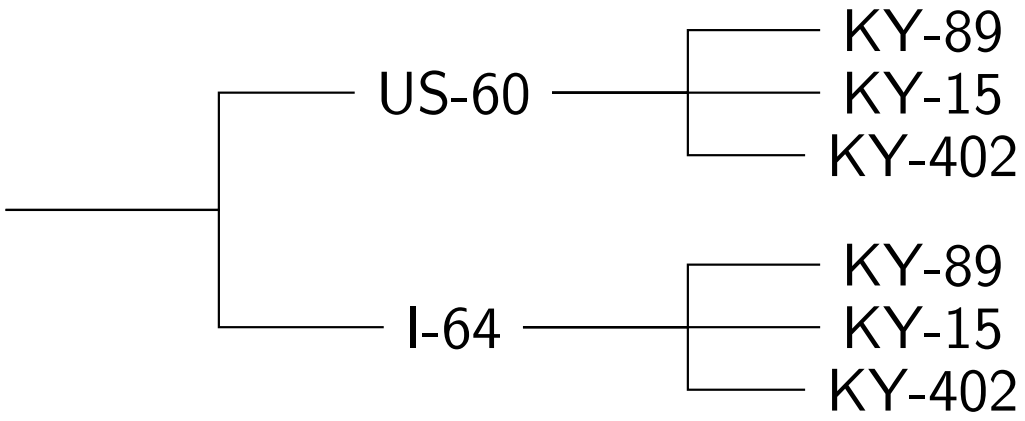

- This is a decision tree. Note how the decision to be made after I-64 is the same as the decision to be made after US-60. The first choice does not affect the second choice. The choices are **independent**.

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- How many cars are in Kentucky?

- 4 million people, about 4 million vehicles, 2 million of which probably have standard plates

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- $(5)(10)(6) \cdot (4)(9)(5) = 54000$.

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  EH, EO, ER, ES, EY,
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- Six possibilities for first (H,O,R,S,E,Y)
  and five for second (the remaining five)