MA162: Finite mathematics

Jack Schmidt

University of Kentucky

November 7, 2012

SCHEDULE:

- HW 6B,6C are due Fri, November 9th, 2012
- Exam 3 is Monday, November 12th, 5pm to 7pm in BS107 and BS116
- Exam 2 grades on Blackboard, PDFs on mathclass

Today we will cover 6.4: Permutations

Exam 3 breakdown

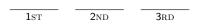
- Chapter 5, Interest and the Time Value of Money
 - Simple interest
 - Compound interest
 - Sinking funds
 - Amortized loans
- Chapter 6, Counting
 - Inclusion exclusion
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 - Multiplication principle
 - Permutations





6.4: Trifecta!

- Some people bet on horse races, a "Trifecta" bet is common
- You predict the first, second, and third place winners, in order.
- There are 14 contenders: Accounting We Will Go, Business
 Planner, Corporate Finance, Debt Sealing, Economy Model, Fiscal
 Filly, Gross Domestic Pony, Horse Resources, Initial Pony Offering,
 Just Another Horsey, Karpay Deeum, LOL Street, Markety Mark,
 and No Chance Vance
- Which ones will you choose? A, B, C or L, N, E?
- How many possibilities?



• There are three places

$$\begin{array}{cccc}
 & 14 & & \\
\hline
 & 1ST & 2ND & 3RD
\end{array}$$

- There are three places
- There are 14 possibilities for first place,

$$\begin{array}{ccc}
14 & 13 \\
\hline
{1ST} & 2{ND} & 3_{RD}
\end{array}$$

- There are three places
- There are 14 possibilities for first place,
- but only 13 left for second place

$$\begin{array}{ccc}
14 & 13 & 12 \\
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- If you bet 1000 times, only a 1 in 3 chance of winning at least once

6.4: Club officers

- The Variety Club has a President, a Vice President, a Secretary, and a Treasurer
- The V.C. has 6 members: Art, Ben, Cin, Dan, Eve, and Fin.
- But every day they want to assign a different set of officers
- Can they make it a year without exactly repeating the officer assignments?
- So maybe ABCD, then ABCE, then ABCF, then ABDC, then . . .

Pres	Vice	Sec.	Trs.

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- There are (6)(5)(4)(3) = 360 possible assignments
- Not enough for a calendar year, but certainly for a school year!

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Pres & Trs.
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- But only five people left for vice president

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 How many ways can a P and a VP be chosen so one is a boy and one is girl?
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$$\begin{array}{ccc}
10 & 5 & = 50 \\
\hline
Pres & Trs.
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- There are ten people eligible for president
- But only five people left for vice president
- That is (5)(10) = 50 different officer assignments

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- So eight for the second part, and six for the third; 10*8*6 = 480 ways.

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- But you don't care what order they are in. So that is four ways:

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4*2 ways counting order, then divide by two to ignore order

6.4: Spelling

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- Well, a little different since there are two Ks
- 8! ways if we keep track of which K is which, then divide by two since each word like KENTUCKY appears twice as kENTUCKY and KENTUCKY.

$$8!/2 = 20160$$

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- Then 5 ways of choosing the goalie.
- Total is: (1365)(165)(56)(5) ways of choosing the first string

6.4: Summary

 We learned to handle symmetries in our counting, especially permutations, and combinations.

 Make sure to complete the homework ASAP, and begin work on the practice exam

Be ready to discuss the practice exam next class; bring a copy