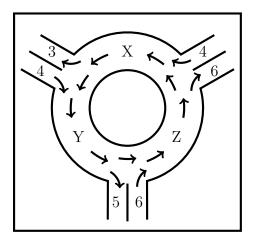Which is busier, X, Y, or Z?

(a) Write equations for the conservation of cars at each of the intersections.

(b) Write the equations as a matrix.

(c) Row reduce the matrix as far as you can, and circle the pivots in the final matrix

(d) Write down the equations corresponding to the REF matrix, and solve them for the circled variables from part (c)

(e) Even if we don't know what Y is, can we tell which road is busiest?

(f) Give a possible set of values for $(X = \underline{\quad}, Y = \underline{\quad}, Z = \underline{\quad})$

(g) Give a different possible set of values for $(X = \underline{\quad}, Y = \underline{\quad}, Z = \underline{\quad})$

(h) Assuming people don't go the wrong way around the round-about (including trying to make immediate U-turns), what is the least busy each section of road could be?

Keep going! Apply row operations to get even more zeros:

$$
\begin{bmatrix} 2 & 1 & 1 & 15 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 1 & 5 \end{bmatrix}
\xrightarrow{R_2-R_3}
\begin{bmatrix} 2 & 1 & 1 & 15 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}
\xrightarrow{R_1-R_3}
\begin{bmatrix} 2 & 1 & 0 & 10 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}
\xrightarrow{R_1-R_2}
\begin{bmatrix} 2 & 0 & 0 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}
\xrightarrow{\frac{1}{2}R_1}
\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{bmatrix}
$$

Now the equations read: $1x = 3$, $1y = 4$, and $1z = 5$.

Now you try some examples:

(a) $\begin{bmatrix} 1 & 3 & 4 & 59 \\ 0 & 1 & 5 & 47 \\ 0 & 0 & 1 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 & 3 & 16 \\ 0 & 1 & -4 & -25 \\ 0 & 0 & 1 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$