MA162: Finite mathematics

Jack Schmidt

University of Kentucky

February 6, 2013

Schedule:

- Exam grades not available until Monday; will be announced when ready
- HW 3.1 due Friday, Feb 08, 2013
- HW 3.2-3.3 due Friday, Feb 15, 2013
- HW 4.1 due Friday, Feb 22, 2013
- HW 2.5-2.6 due Friday, Mar 01, 2013
- Exam 2, Monday, Mar 04, 2013, from 5pm to 7pm

Today we will cover 3.1: graphing linear inequalities

Exam 2: Overview

- Ch. 3, Linear optimization with 2 variables
  1. Graphing linear inequalities
  2. Setting up linear programming problems
  3. Method of corners to find optimum values of linear objectives
- Ch. 4, Linear optimization with millions of variables
  1. Slack variables give us flexibility in RREF
  2. Some RREFs are better (business decisions) than others
  3. Simplex algorithm to find the best one using row ops
  4. Accountants and entrepreneurs are two sides of the same coin
- Chapter 2 high-level view
  1. Composition of business processes
  2. Changing resource levels

3.1: Inequalities

- Xylophones cost \$200 each and Yukuleles cost \$100 each
- Your need instruments for your new band Glük-N-Spiel
- Your insane and rich uncle only gave you a budget of \$1000
- What are your options?

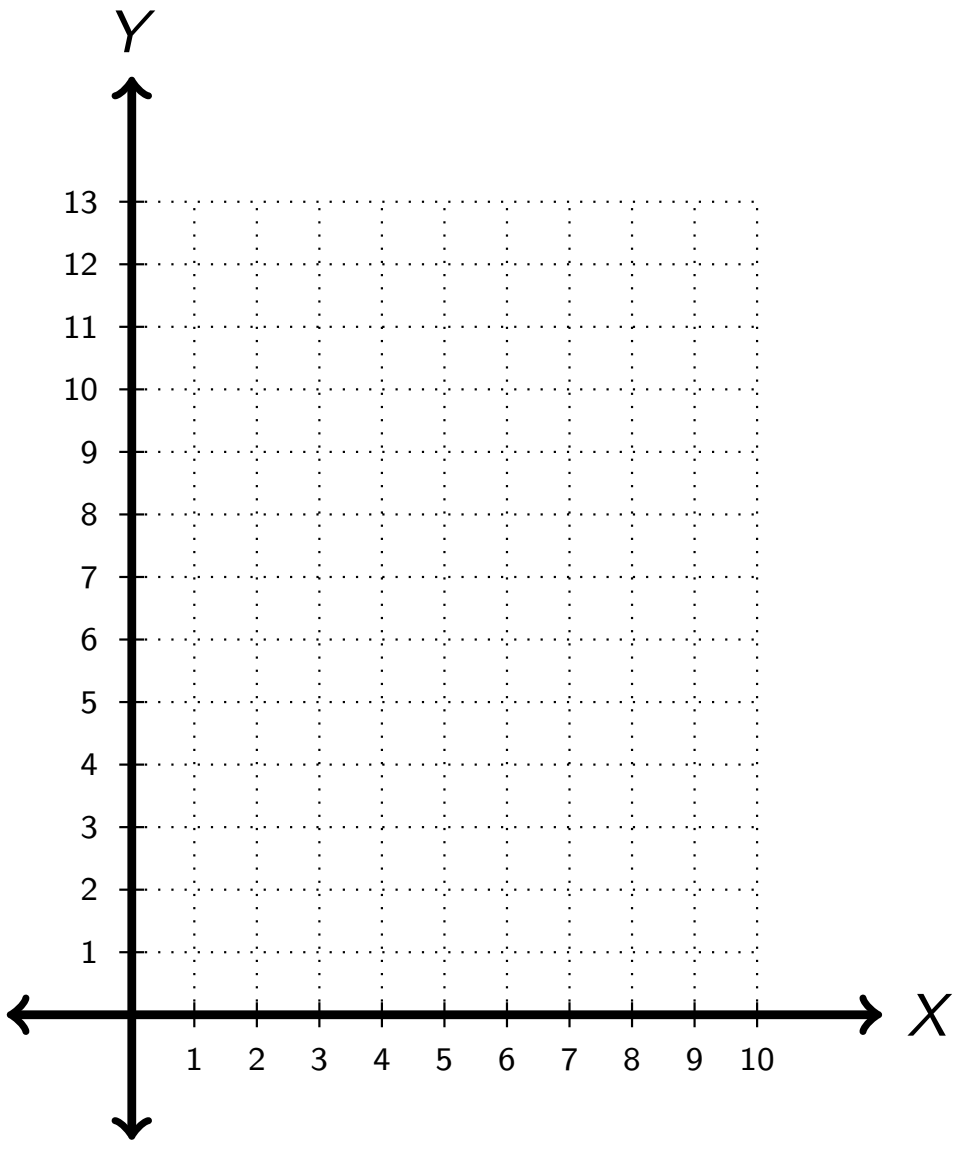
$$200x + 100y = 1000$$

3.1: Inequalities

- Xylophones cost \$200 each and Yukuleles cost \$100 each
- Your need instruments for your new band Glük-N-Spiel
- Your insane and rich uncle only gave you a budget of \$1000
- What are your options? Don't have to spend it all!

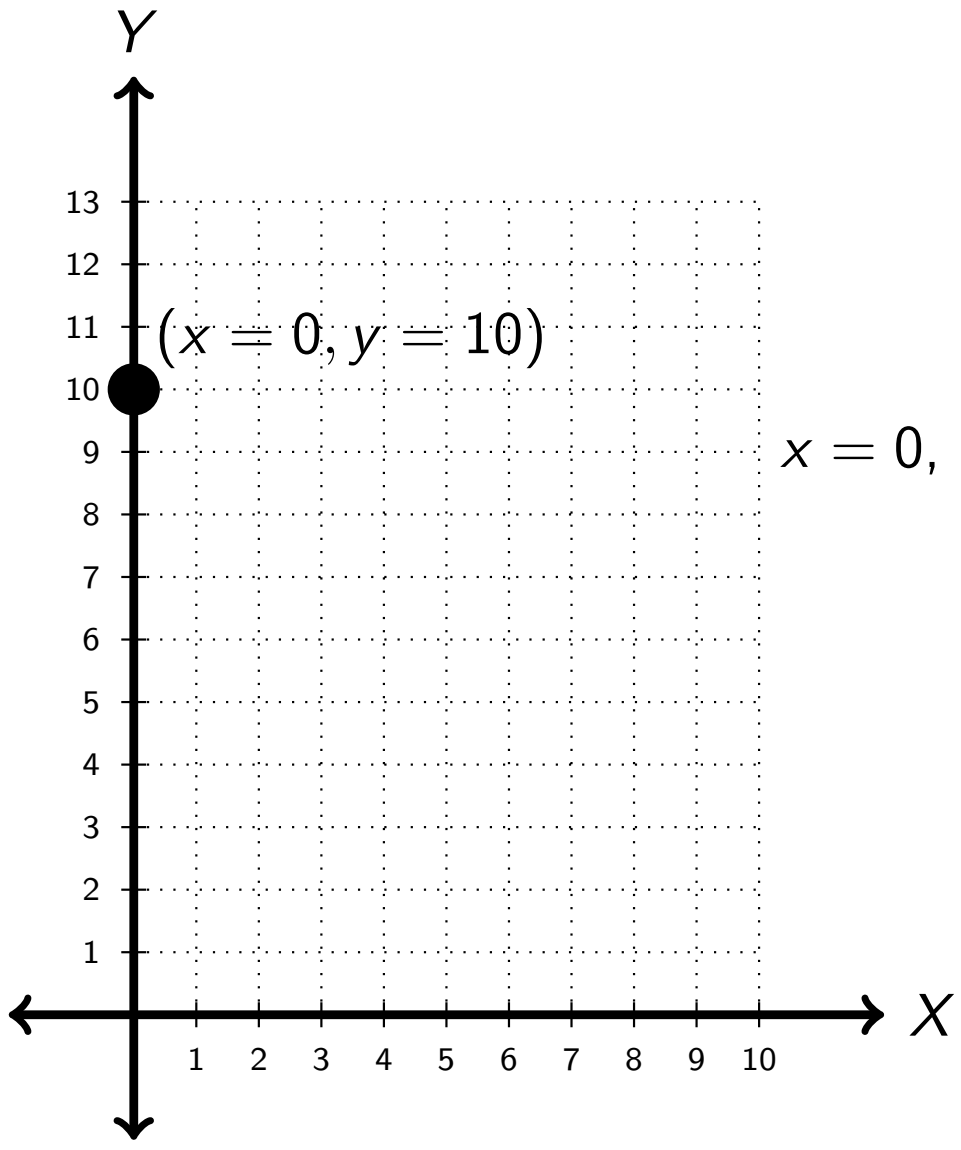
$$200x + 100y \leq 1000$$

3.1: Graphing inequalities



$$200x + 100y = 1000$$

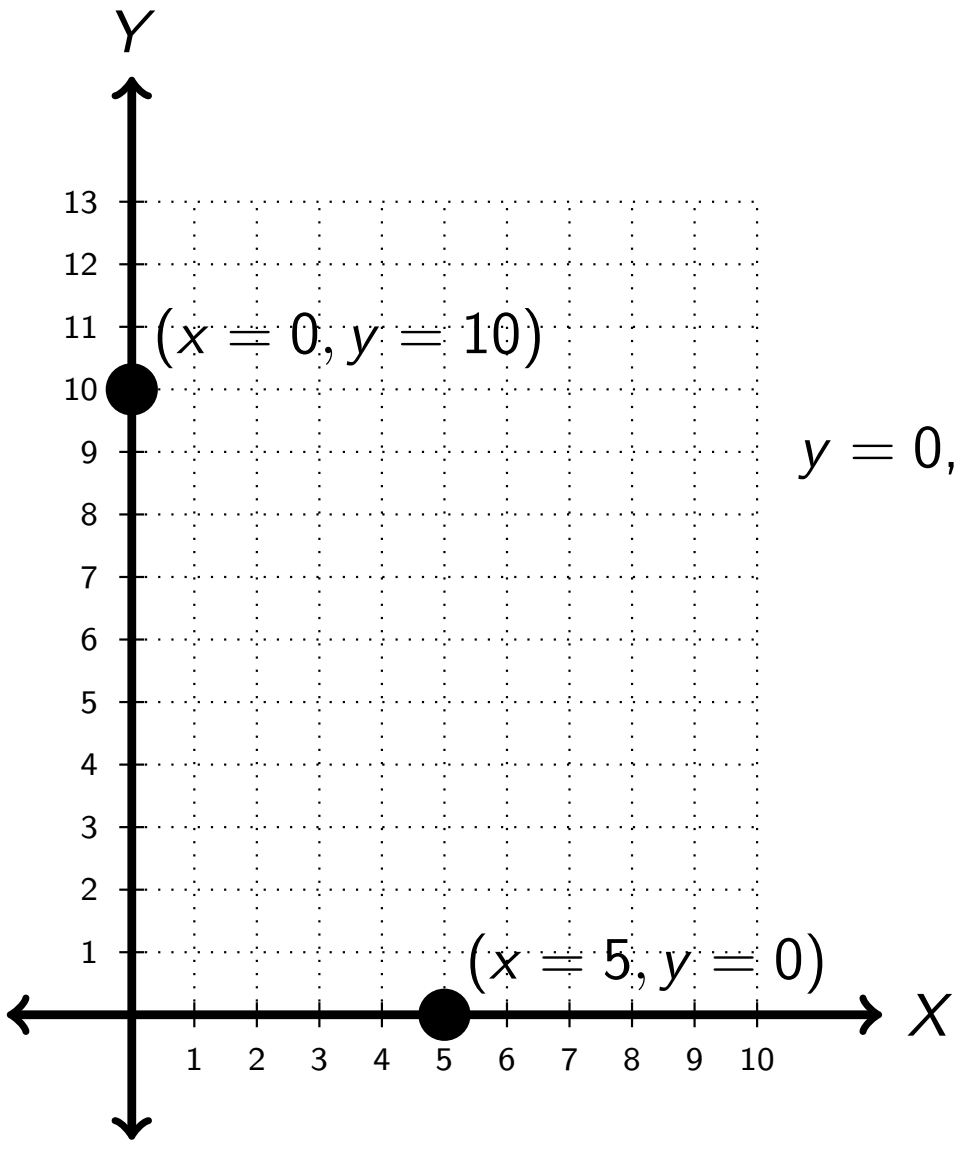
3.1: Graphing inequalities



$200x + 100y = 1000$

$x = 0,\ 100y = 1000,\ y = 10$

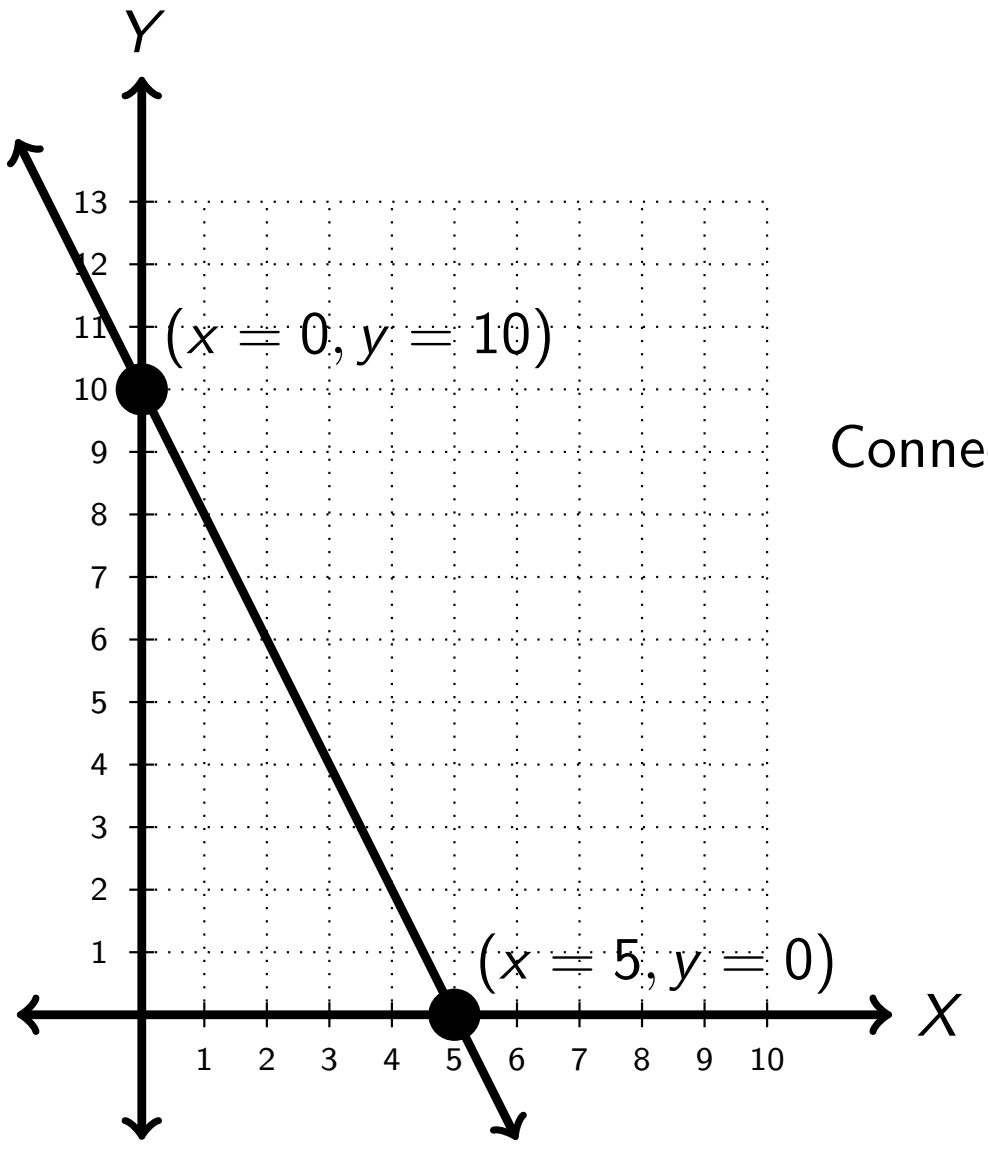
3.1: Graphing inequalities



$$200x + 100y = 1000$$

$$y = 0,\ 200x = 1000,\ x = 5$$

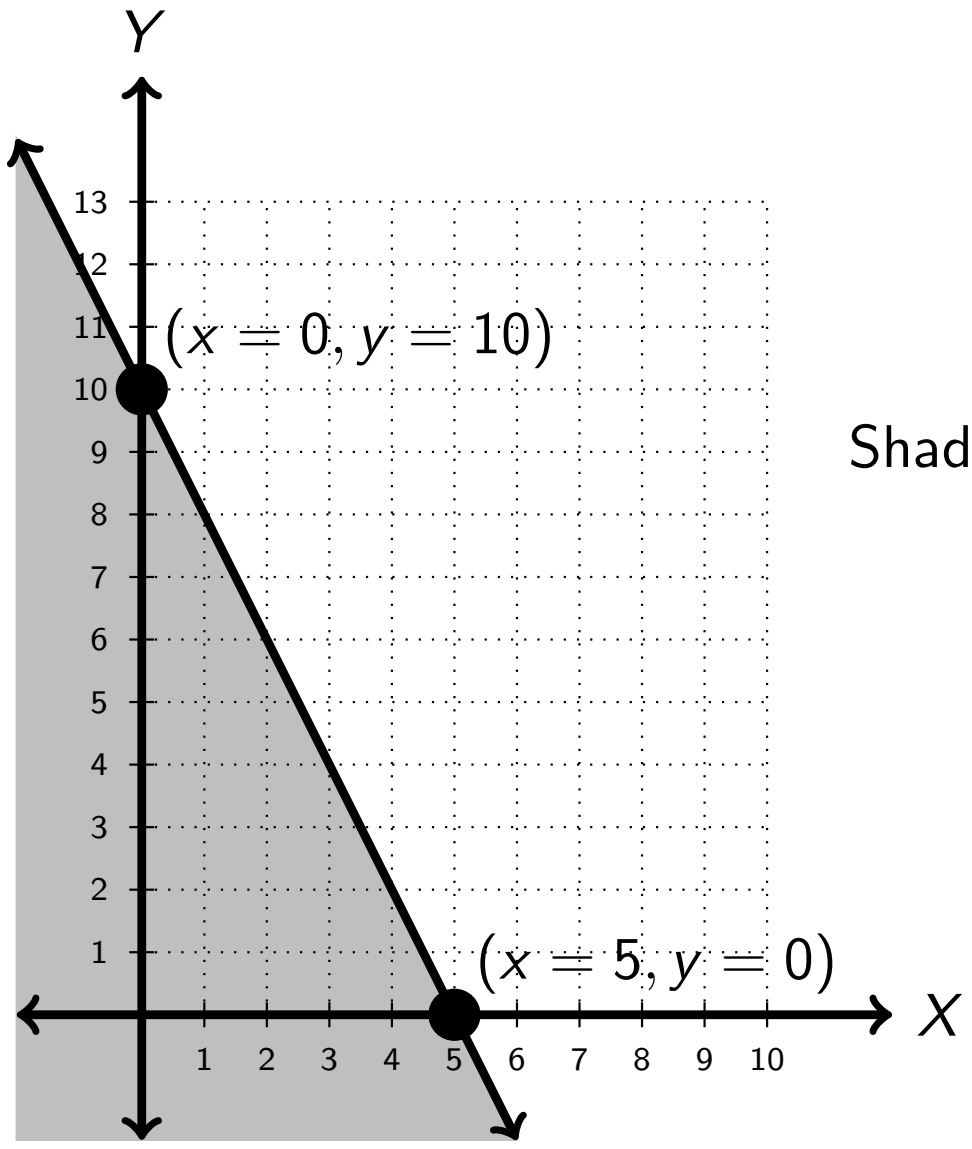
3.1: Graphing inequalities



$200x + 100y = 1000$

Connect the dots

3.1: Graphing inequalities



$200x + 100y \leq 1000$

Shade the region

3.1: Graphing inequalities

- First graph the "equality", that is, graph the line

$\Rightarrow$ Find two points on the line and then draw the connection

- Next graph the inequality, that is, shade the region

$\Rightarrow$ Choose a point not on the lines and see if it is on the correct side

- For example (0,0) is on the correct side since

$$(200)(0) + (100)(0) \leq 1000$$

3.1: Is it realistic?

- Our region is very large.
- Some points don't make sense for a single purchaser:

$\Rightarrow$ $(2.5, 3.5)$ means buy 2.5 Xylophones and 3.5 Yukuleles ($850)

- But maybe it makes sense as an average or a strategy
- Some points don't make any sense for any purchaser:

$\Rightarrow$ $(-10, -20)$ means buy -10 Xylophones . . . (-$4000)

3.1: Systems of inequalities

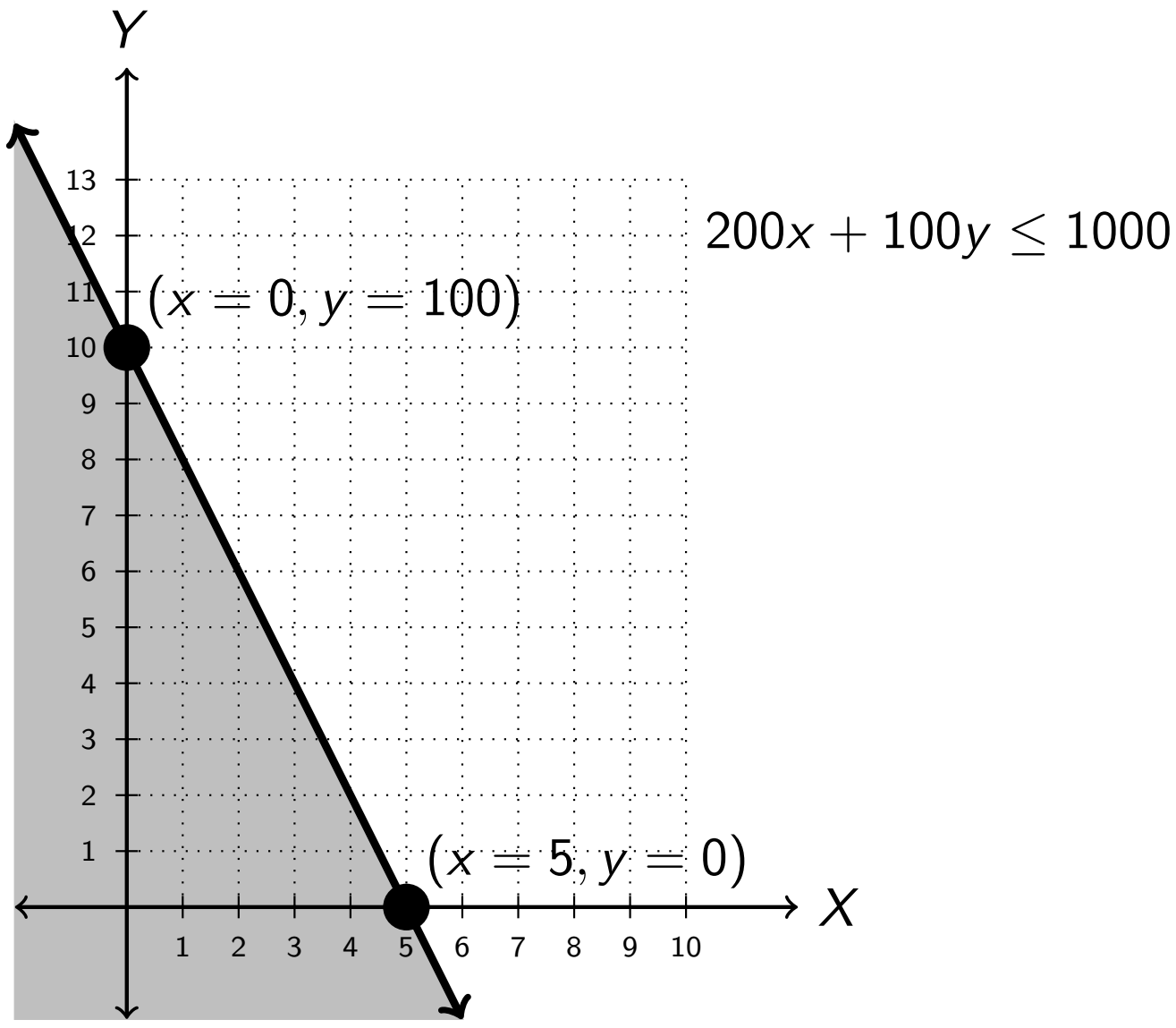
- We also need some sanity: $X \geq 0$ and $Y \geq 0$
- So we have a system of inequalities:

$$\begin{cases} 200X + 100Y \leq 1000 \\ X \geq 0, Y \geq 0 \end{cases}$$

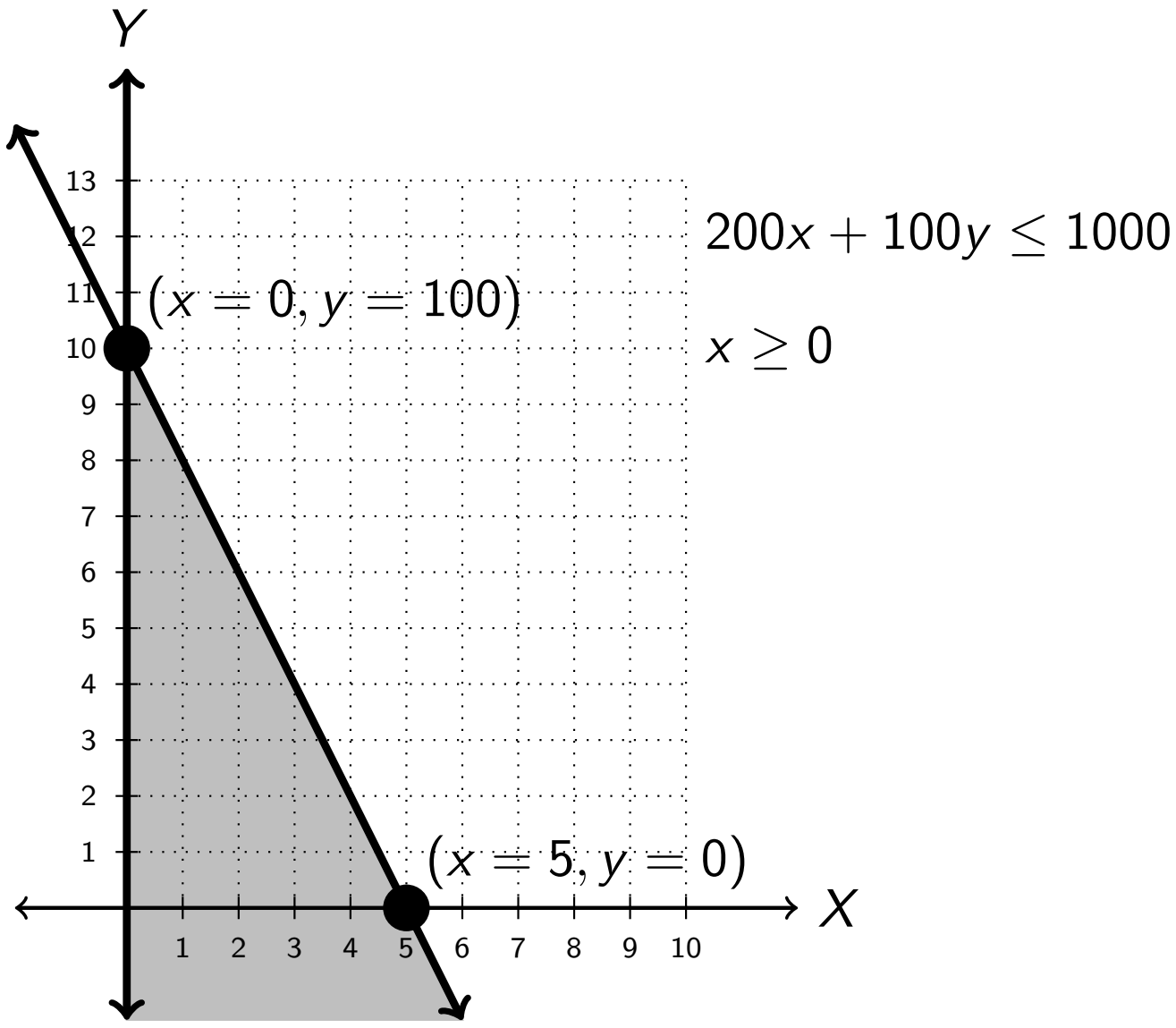
- Not enough for just one to be true!

$\Rightarrow$ $(500, 0)$ would be very expensive ($100,000) and noisy!

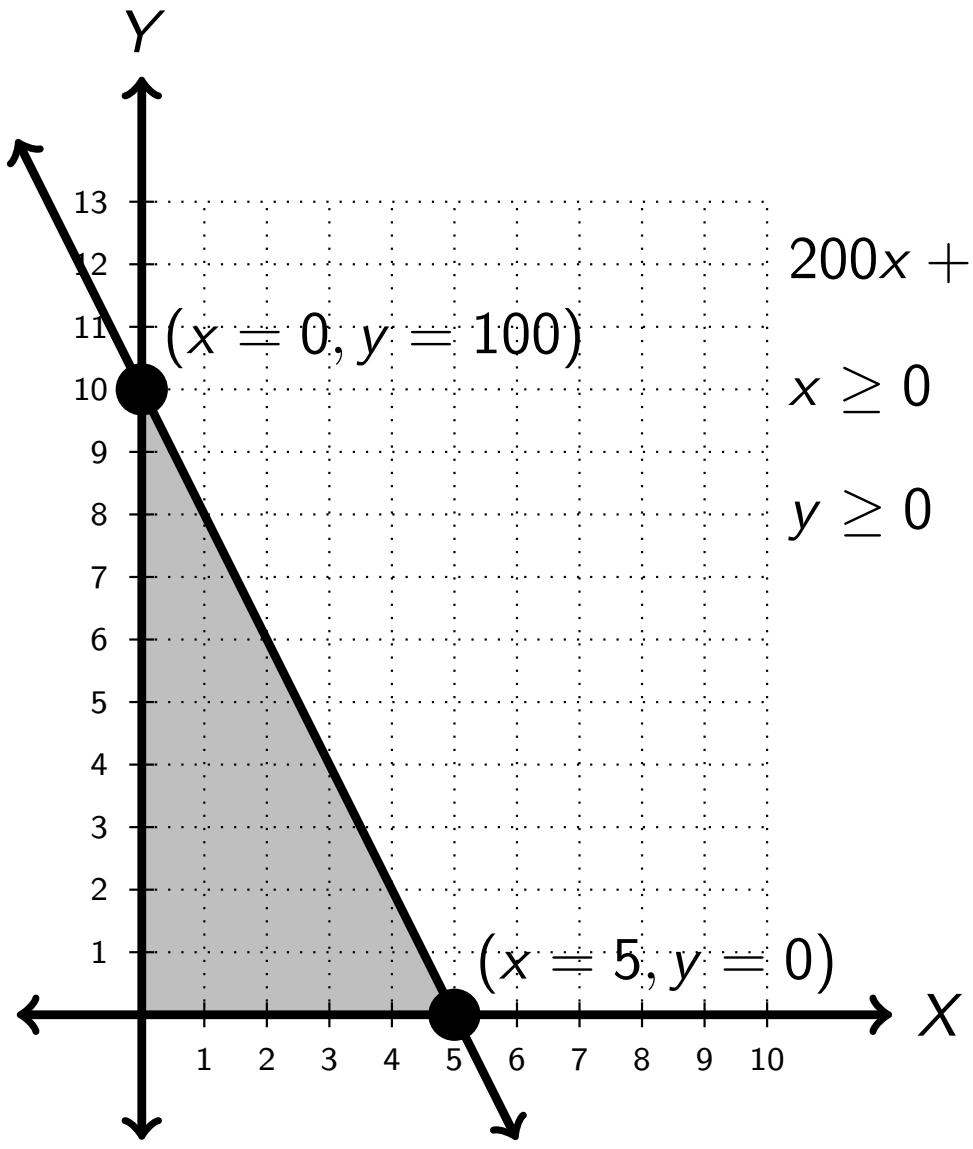
3.1: Graphing systems of inequalities



3.1: Graphing systems of inequalities



3.1: Graphing systems of inequalities



$200x + 100y \leq 1000$

$x \geq 0$

$y \geq 0$

3.1: Graphing systems of inequalities

- Graph each equality (line)
- Figure out which side of the line is good
- Shade the region that is on the correct side of **all** lines
- Alternatively: figure out which of the pieces is good

Jellybeans and Chocolates

- Suppose each bag of jelly beans costs \$1,
  each bar of chocolate costs \$2
- We have \$100 to spend, and plan on buying J bags and C bars
- If we want to stay under budget, then we need $J + 2C \leq 100$
- Maybe I like chocolate more than colored sugar,
  and I want 500 bars of chocolate.
- Setting $C = 500$, we solve $J \leq 100 - 2C = 100 - 1000 = -900$
- Ok, I just need to buy -900 bags of Jelly beans.

3.1: Our standard inequalities

- In a realistic scenario we cannot "sell jelly beans short"
- Not only must we stay under budget, $J + 2C \leq 100$,
- We must also stay sane: $J \geq 0$, $C \geq 0$
- These are the **standard inequalities** are are almost always in effect
- Are these the only inequalities affecting us?

3.1: More inequalities

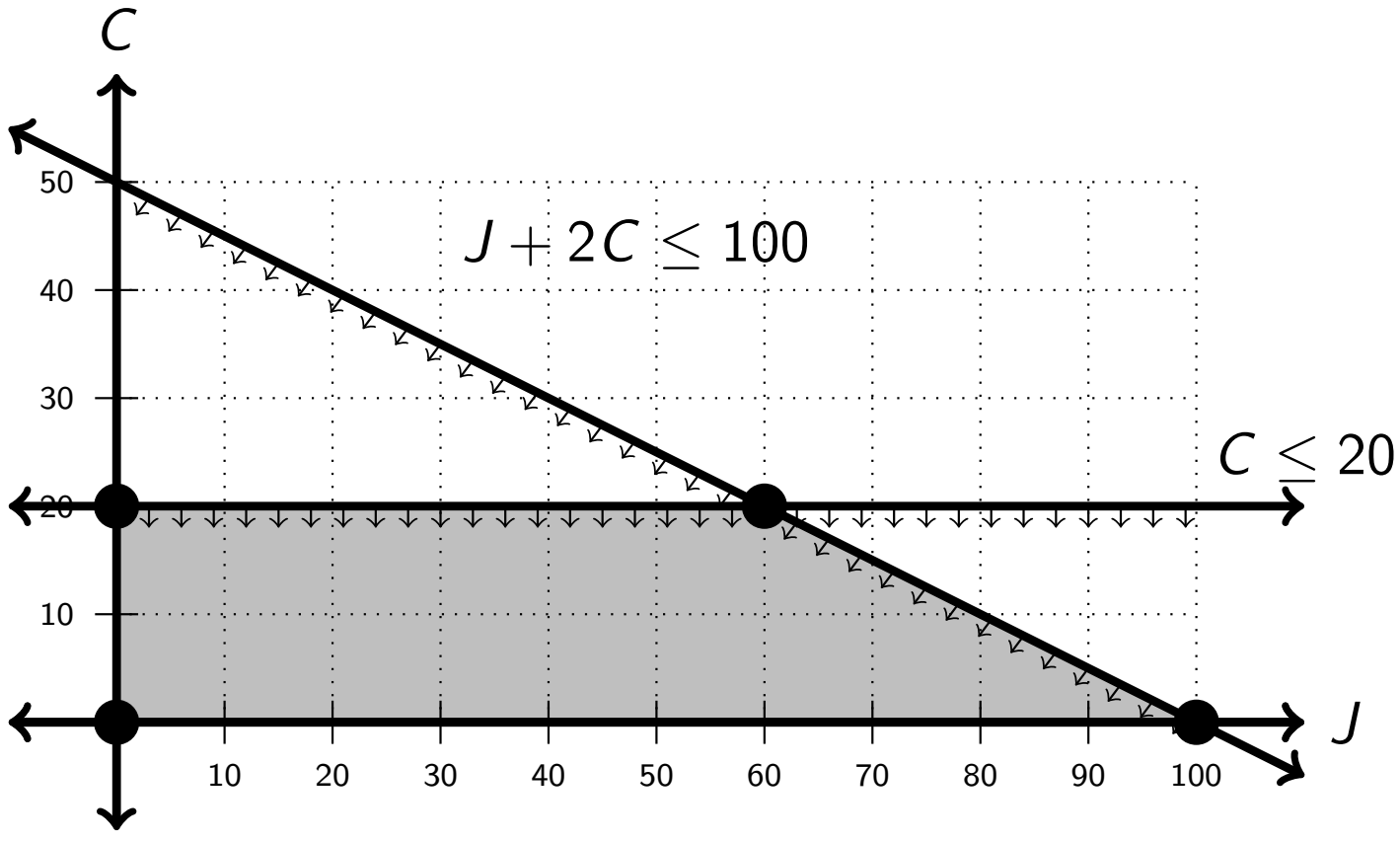
- Unfortunately, the local convenience store is run by old man Charlie
- Charlie hates chocolate bars and barely stocks them
- Today he only has 20 bars
- Apparently we have a new inequality: $C \leq 20$
- All told that is:

$$J + 2C \leq 100$$
$$C \leq 20$$
$$J \geq 0, C \geq 0$$

3.1: Summarizing the inequalities

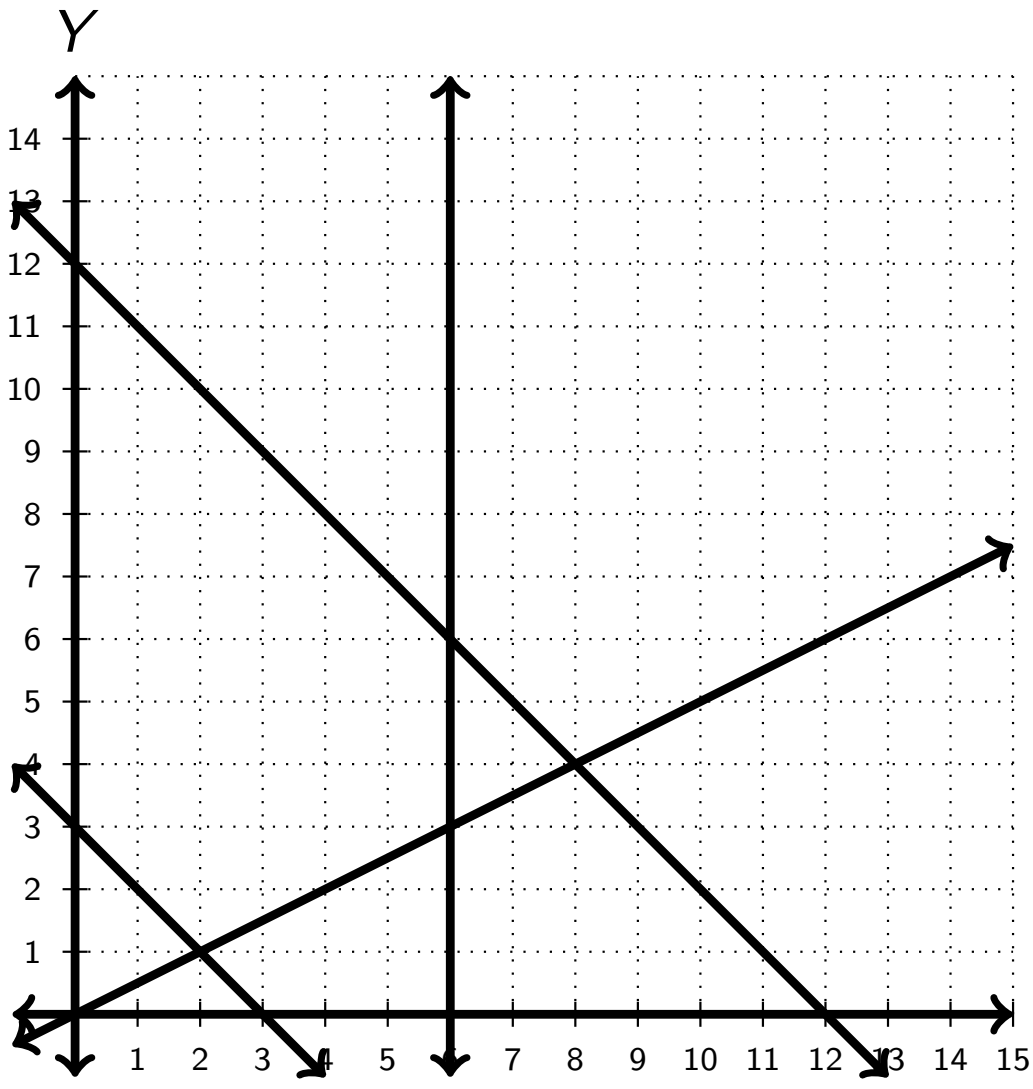
- It is a little tricky to summarize the possibilities:
- We could buy nothing, $(J = 0, C = 0)$
- We could buy all the chocolate, $(J = 0, C = 20)$,
- and we could spend the rest of the money on jelly beans $(J = 60, C = 20)$
- We could buy only jelly beans $(J = 100, C = 0)$
- If we actually need to bring a bunch of candy, it's not really clear whether we should do one of these extremes or something in between

3.1: Graphing systems of inequalities



Draw little arrows to show which side is good.

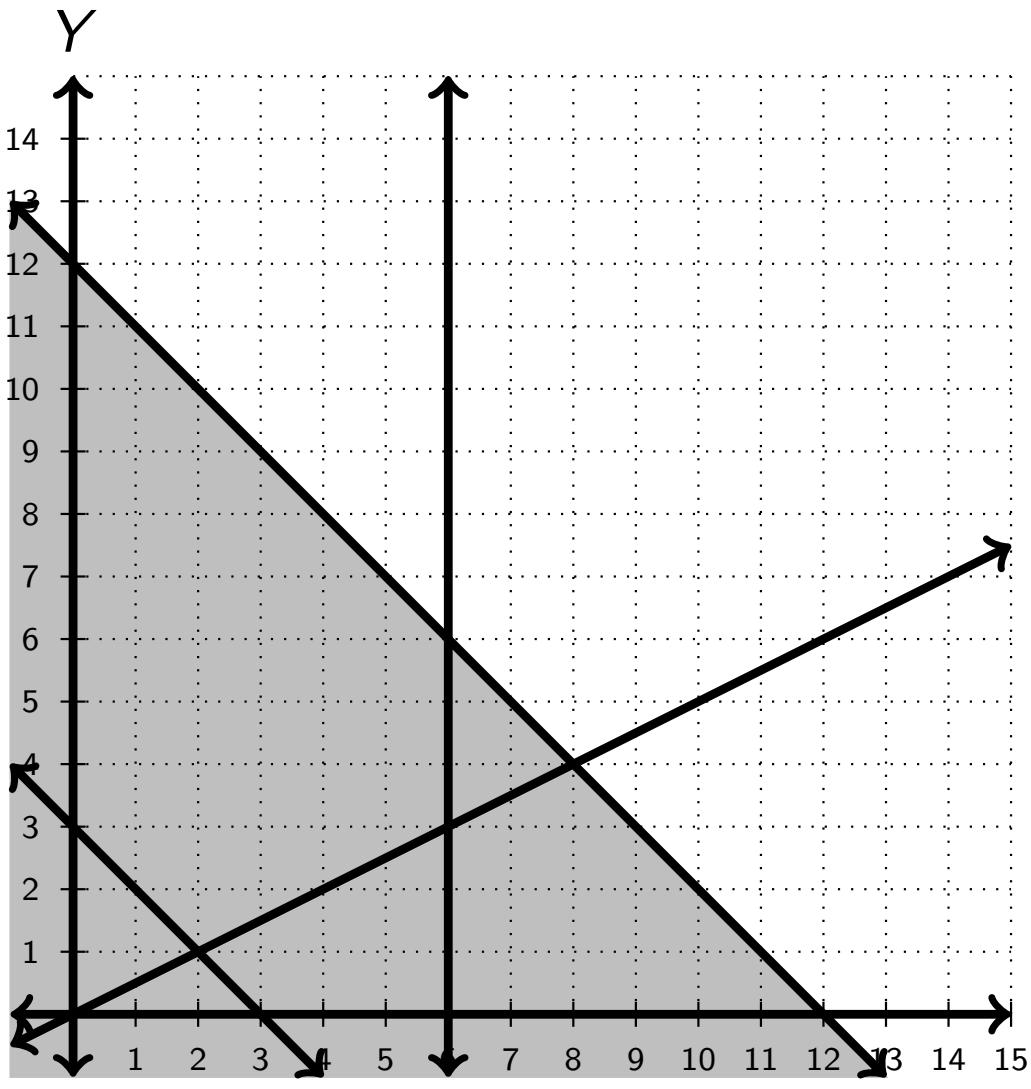
3.1: Graphing systems of inequalities



$$\begin{cases} x + y = 12 \\ x - 2y = 0 \\ x + y = 3 \\ x = 6 \\ x = 0 \\ y = 0 \end{cases}$$

Draw all the lines, then check each inequality.

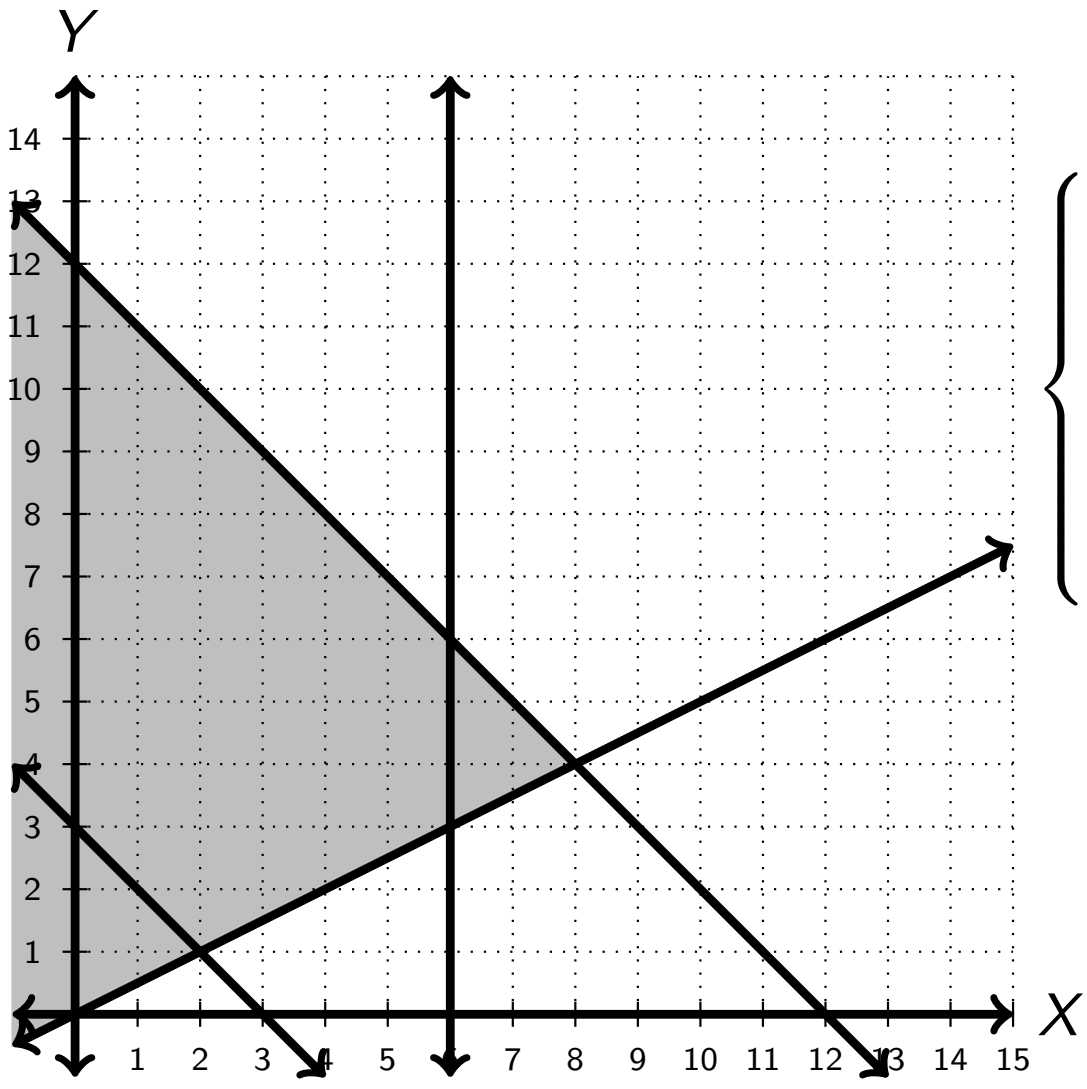
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y = 0 \\ x + y = 3 \\ x = 6 \\ x = 0 \\ y = 0 \end{cases}$$

Draw all the lines, then check each inequality.

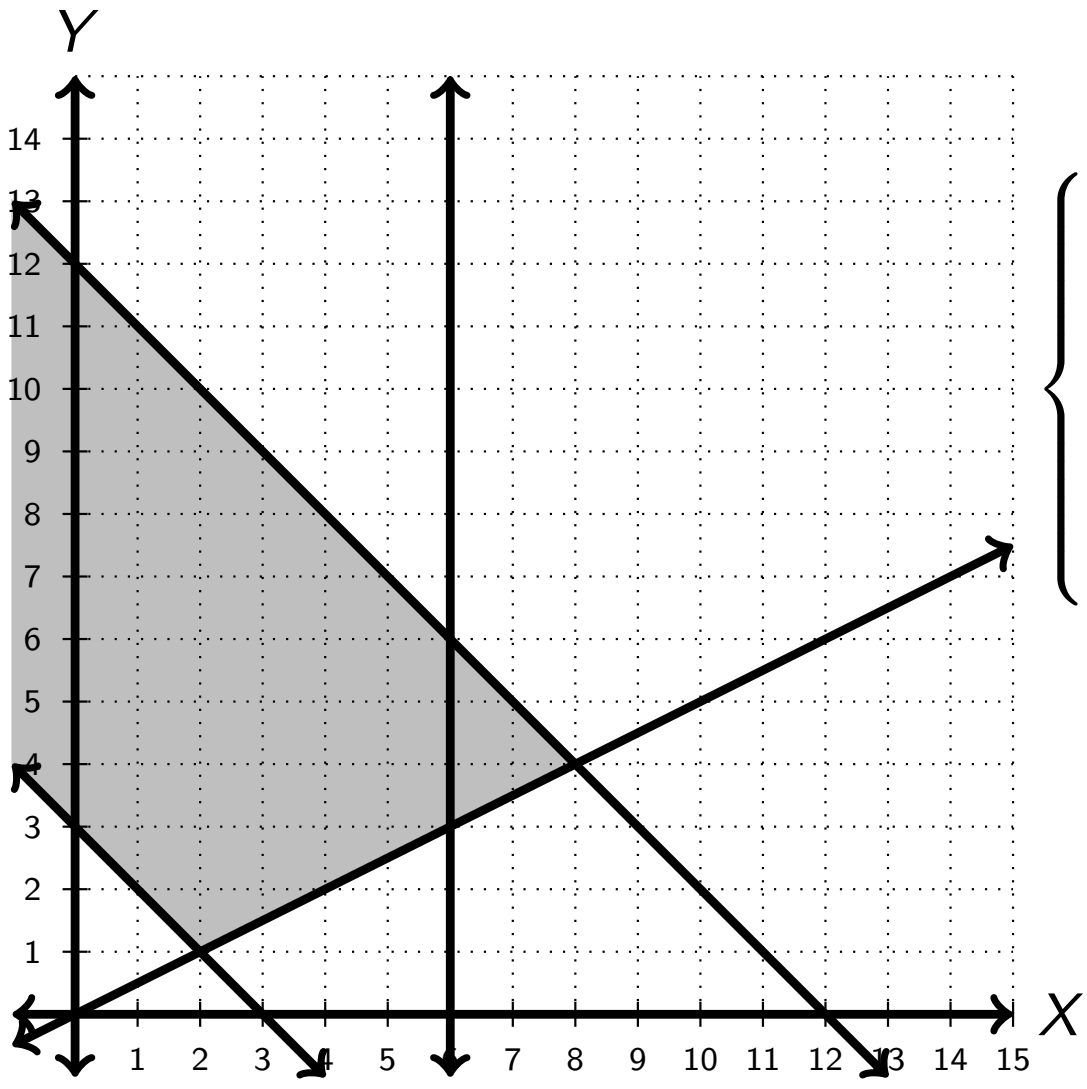
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y = 3 \\ x = 6 \\ x = 0 \\ y = 0 \end{cases}$$

Draw all the lines, then check each inequality.

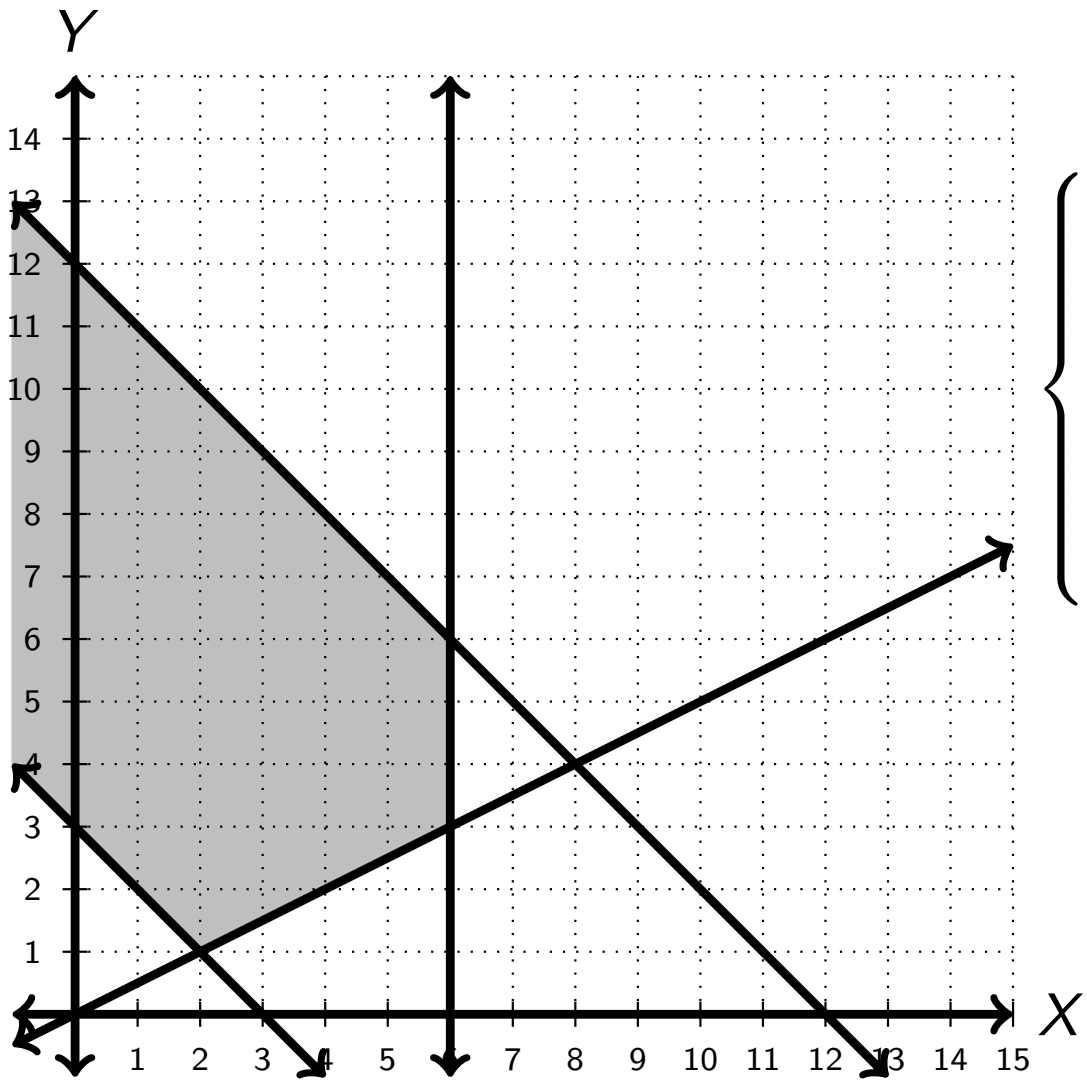
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x = 6 \\ x = 0 \\ y = 0 \end{cases}$$

Draw all the lines, then check each inequality.

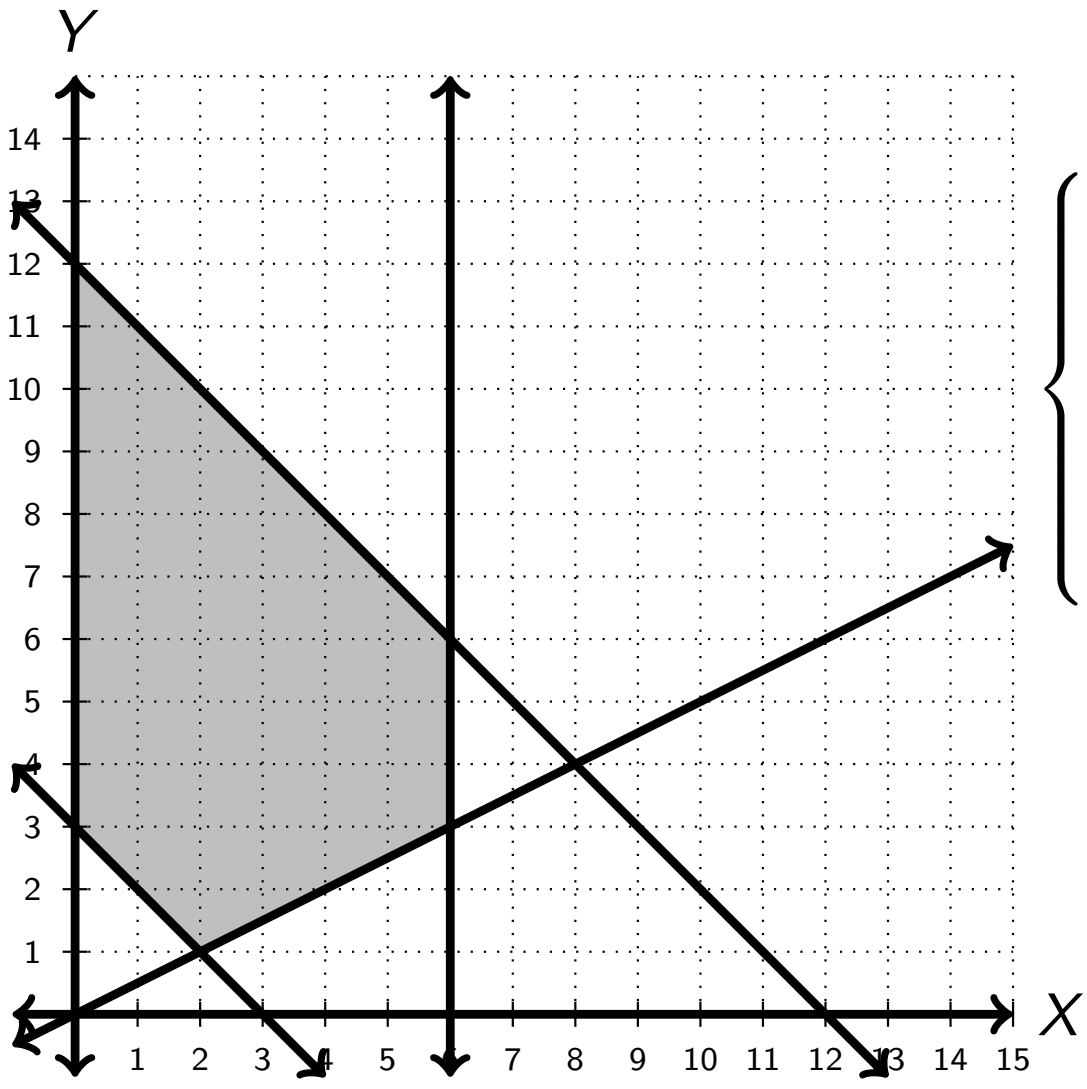
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x = 0 \\ y = 0 \end{cases}$$

Draw all the lines, then check each inequality.

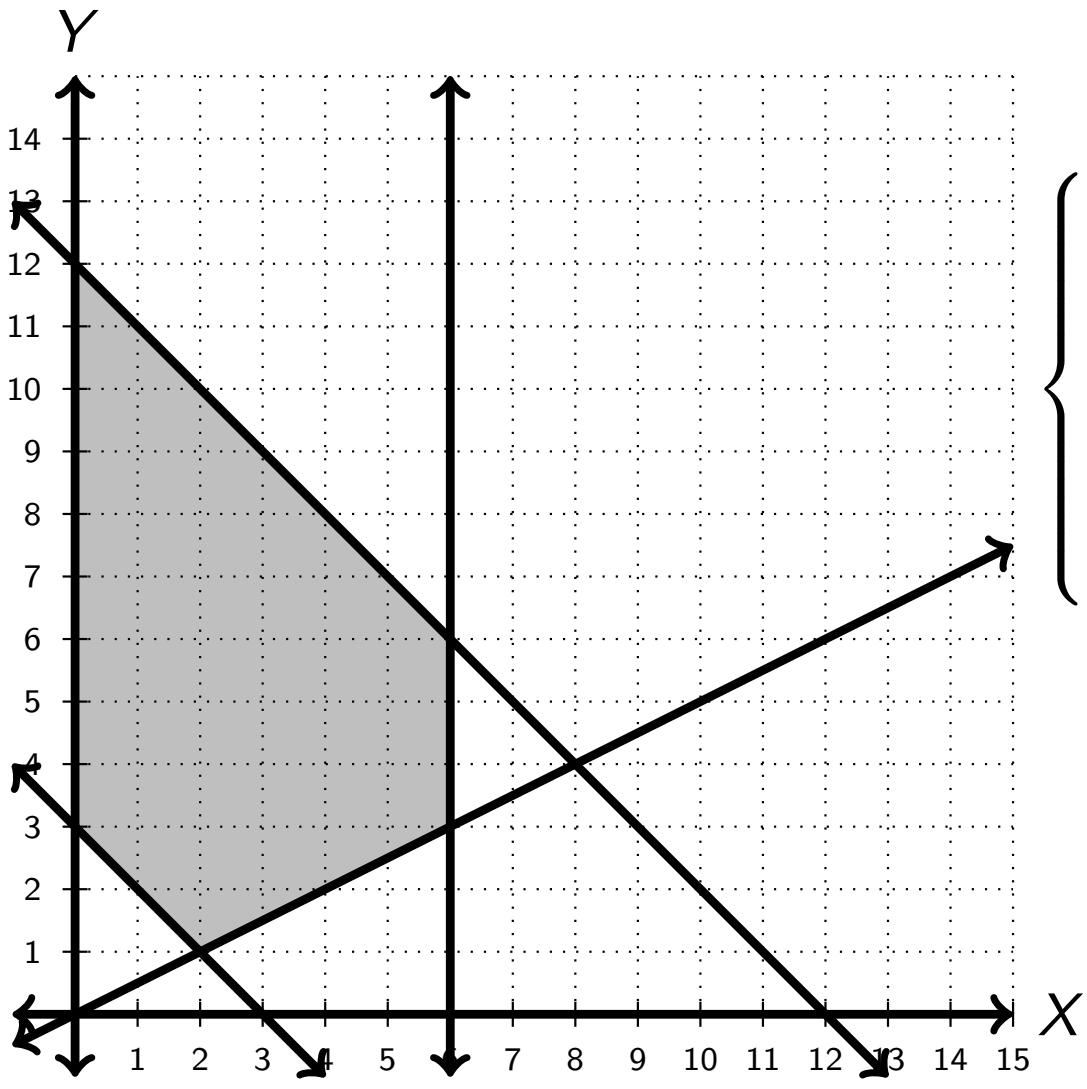
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y = 0 \end{cases}$$

Draw all the lines, then check each inequality.

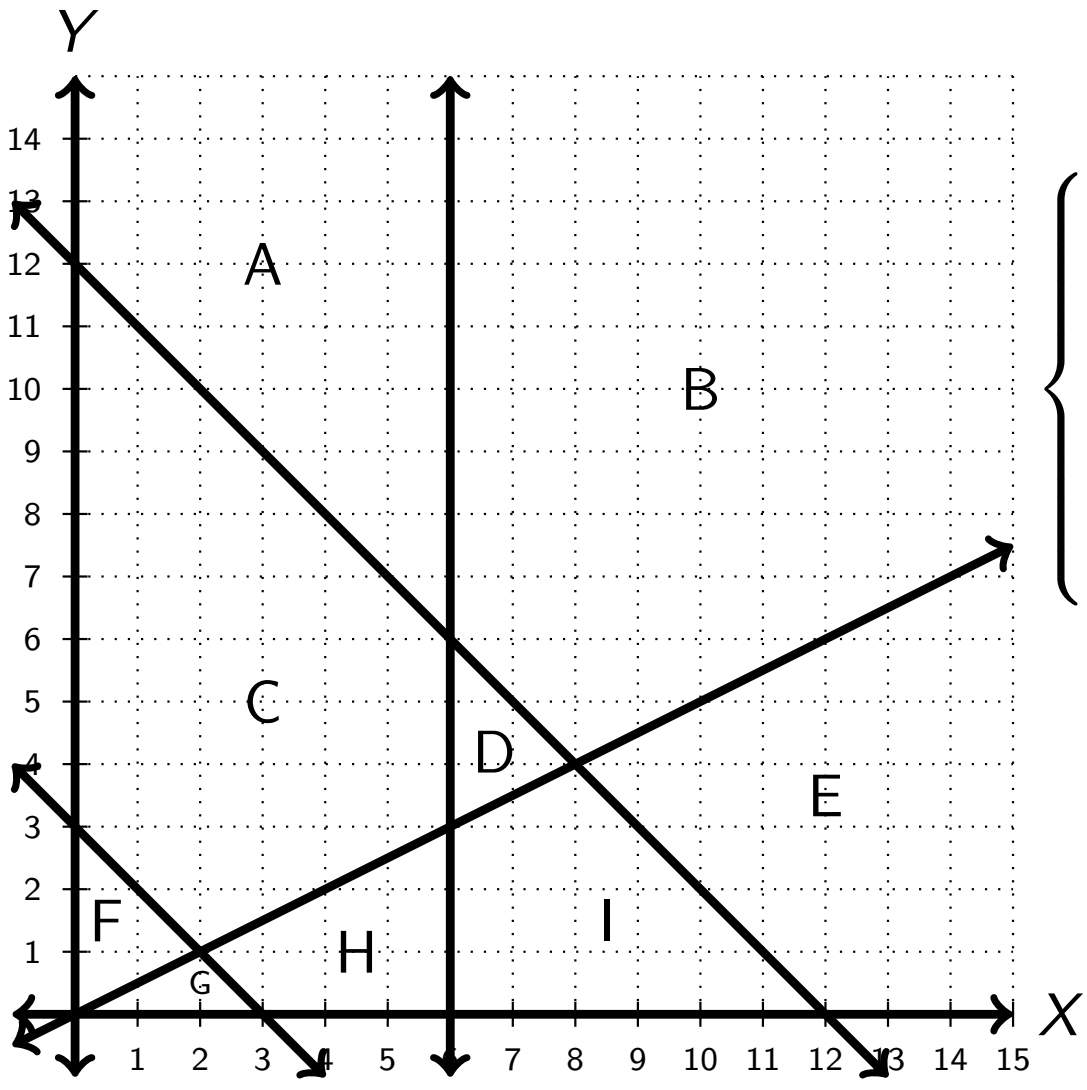
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Draw all the lines, then check each inequality. Too many regions!

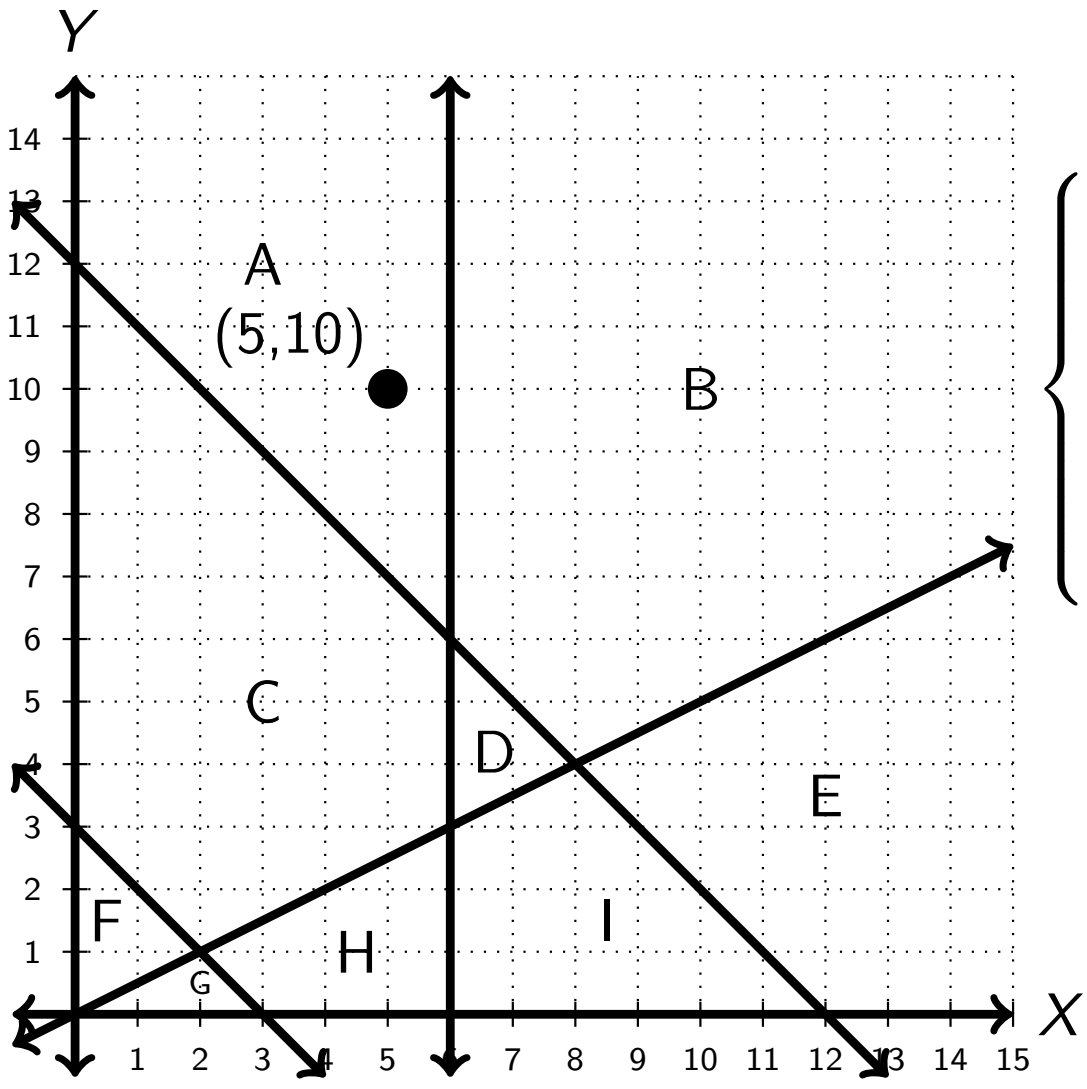
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Check a point in each region to find the right one.

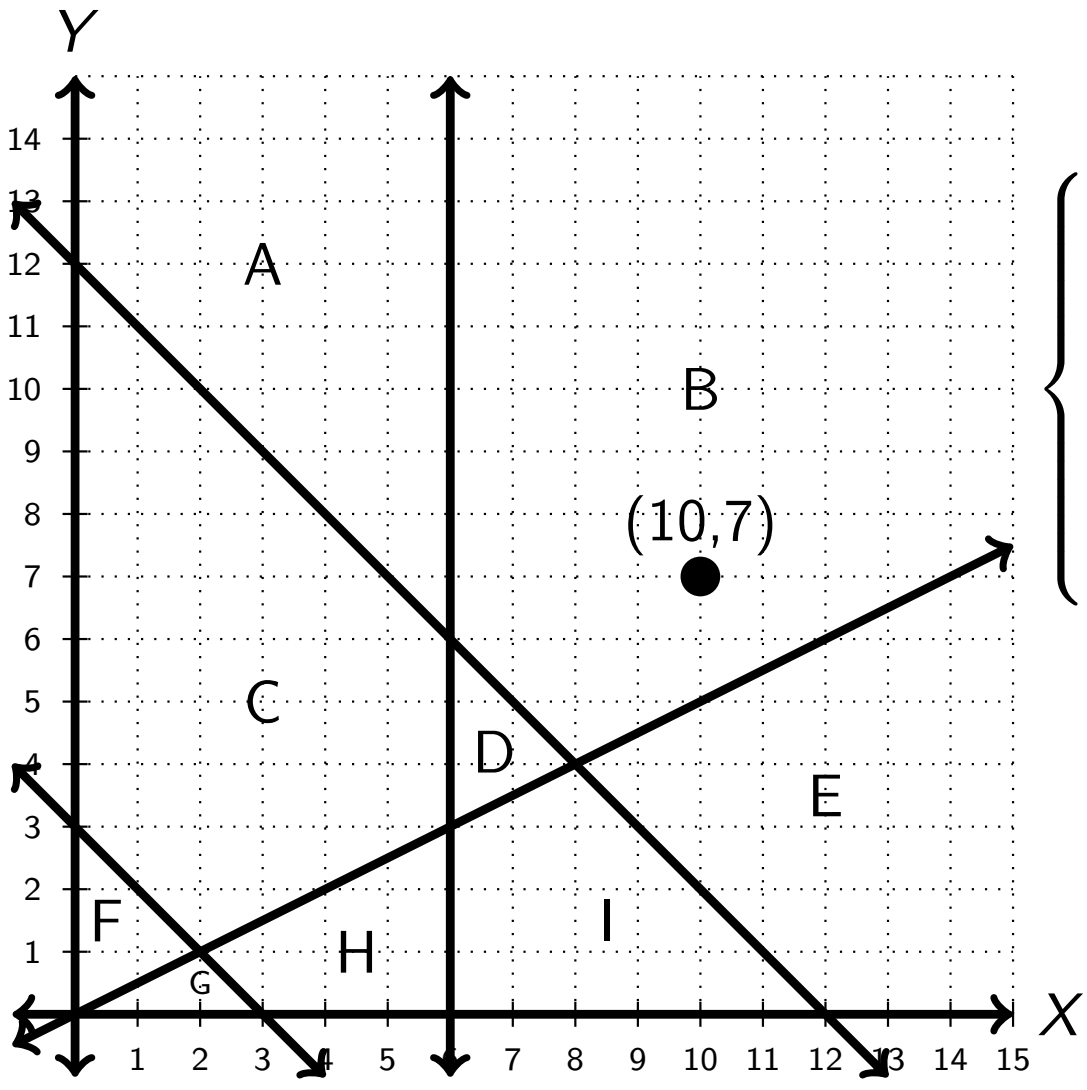
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Check a point in each region to find the right one.

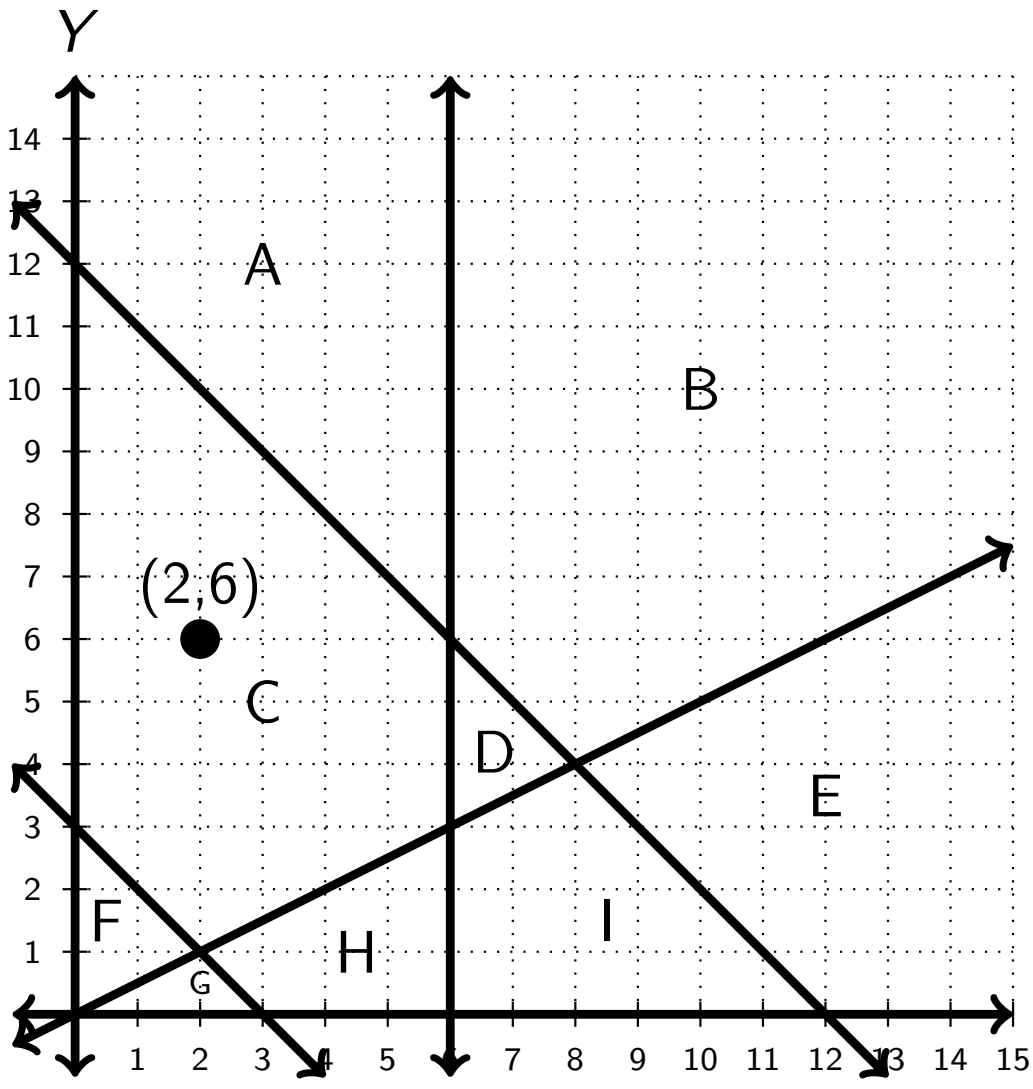
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Check a point in each region to find the right one.

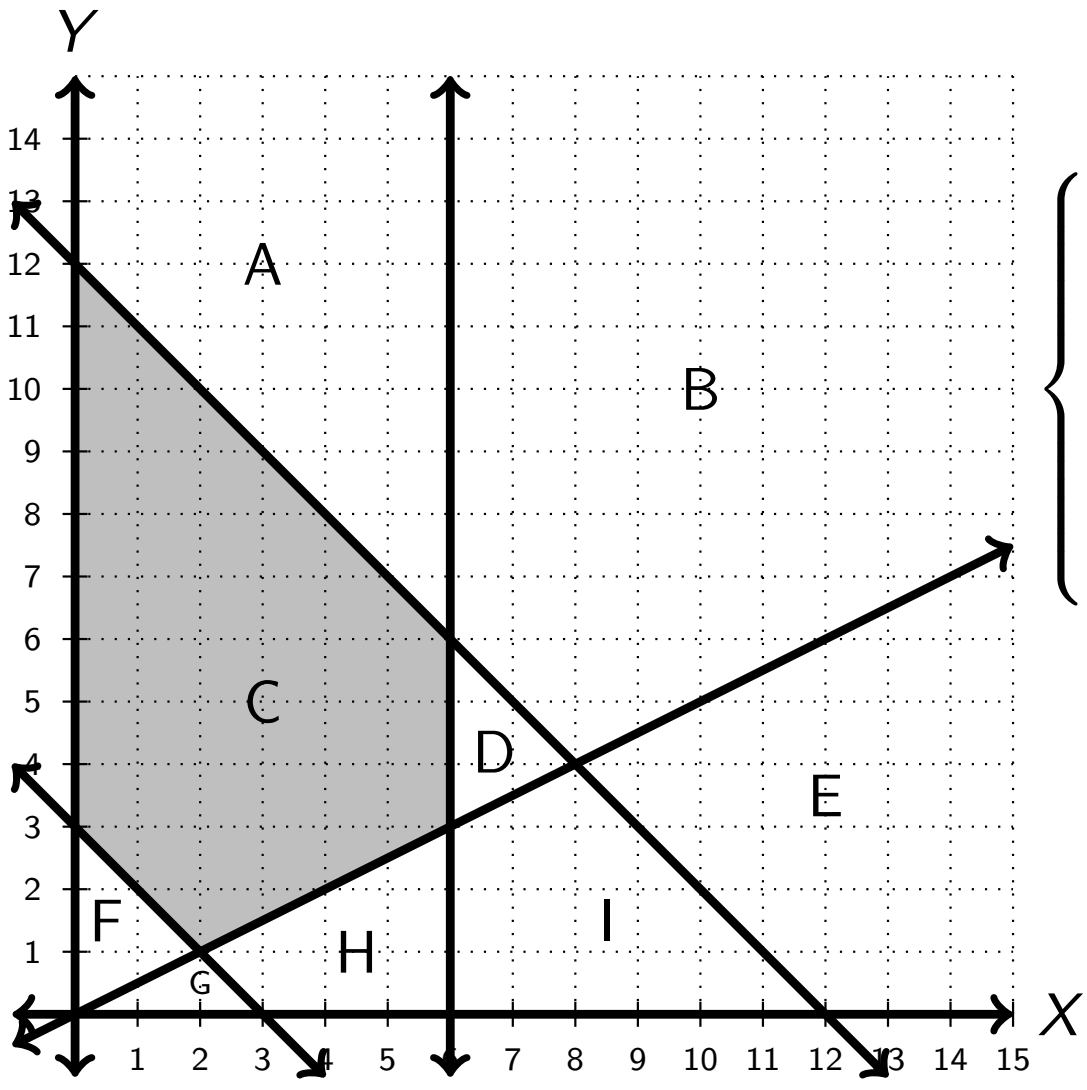
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Check a point in each region to find the right one.

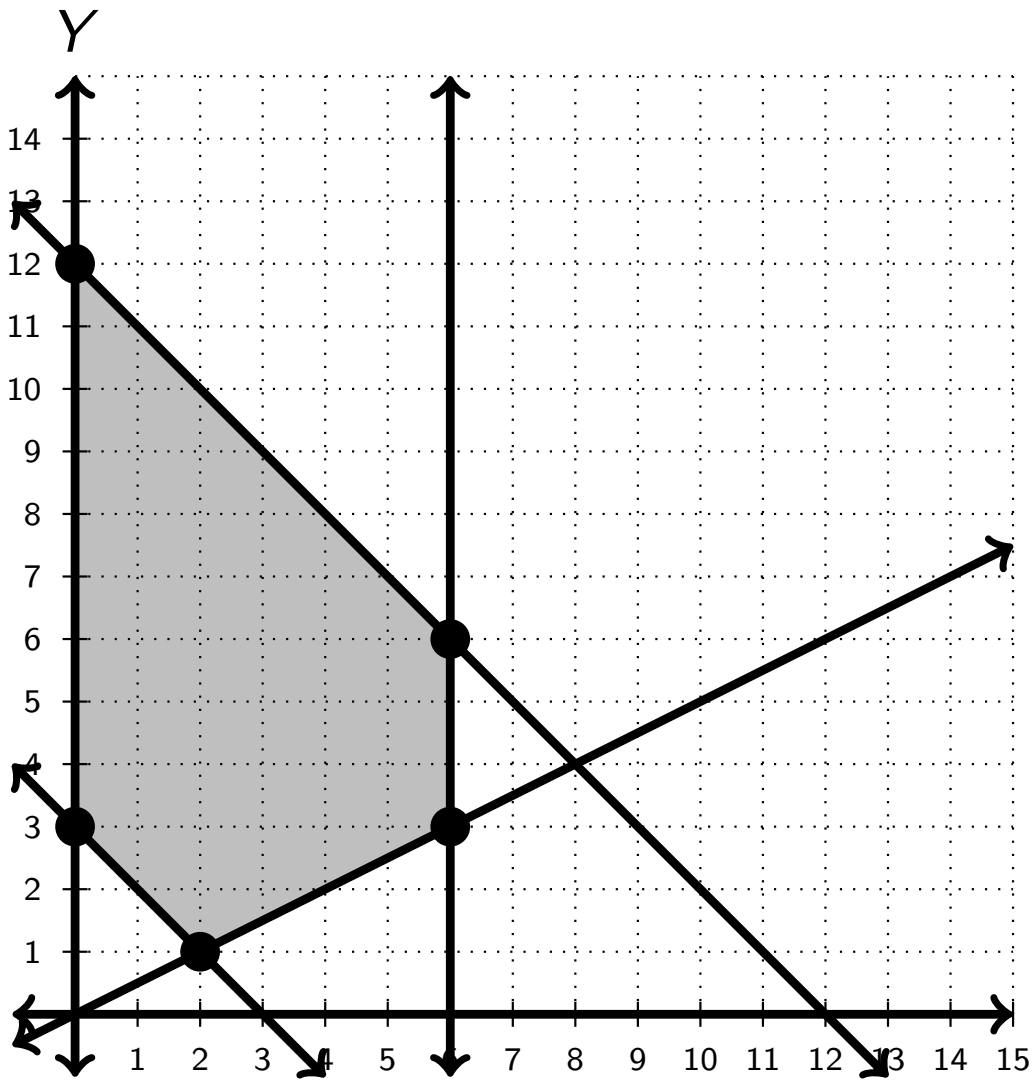
3.1: Graphing systems of inequalities



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Check a point in each region to find the right one. Yay!

3.1: Finding corners



$$\begin{cases} x + y \leq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Intersect each pair of lines, and check it satisfies other inequalities

3.1: Finding corners

- For each pair of lines, find the intersection
- Then check that intersection satisfies the rest of the inequalities
- Not all intersections are corners!
- All corners are intersections.
- Intersections are just 2×3 RREF problems!

3.1: How many corners are there?

- How many angles does a triangle have?

3.1: How many corners are there?

- How many angles does a triangle have?
- How many sides does a triangle have?

3.1: How many corners are there?

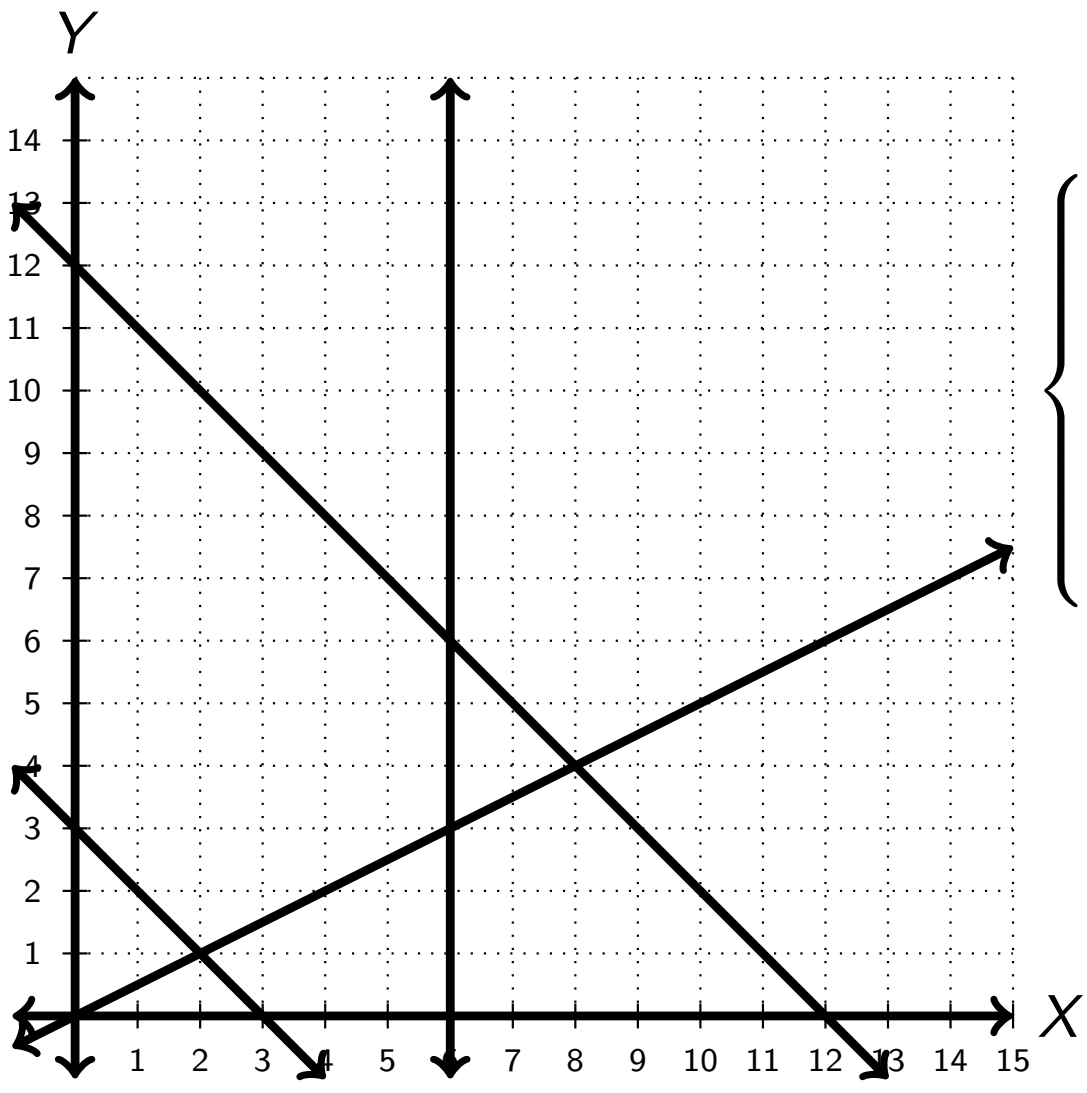
- How many angles does a triangle have?
- How many sides does a triangle have?
- How many angles does a quadrangle have? A quadrilateral?

3.1: How many corners are there?

- How many angles does a triangle have?
- How many sides does a triangle have?
- How many angles does a quadrangle have? A quadrilateral?
- An n-sided polygon has n angles too!

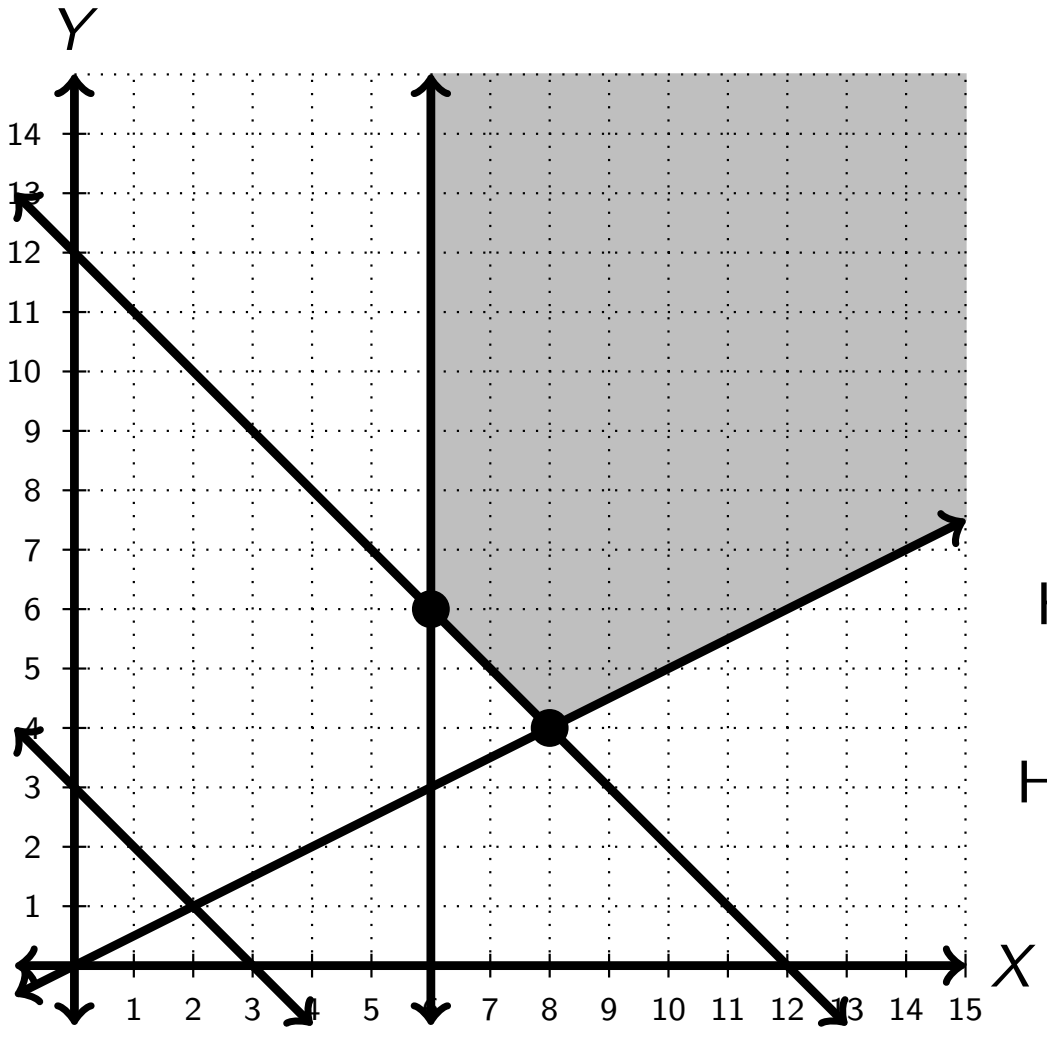
3.1: Worksheet #4

- Now you try!



$$\begin{cases} x + y \geq 12 \\ x - 2y \leq 0 \\ x + y \geq 3 \\ x \geq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

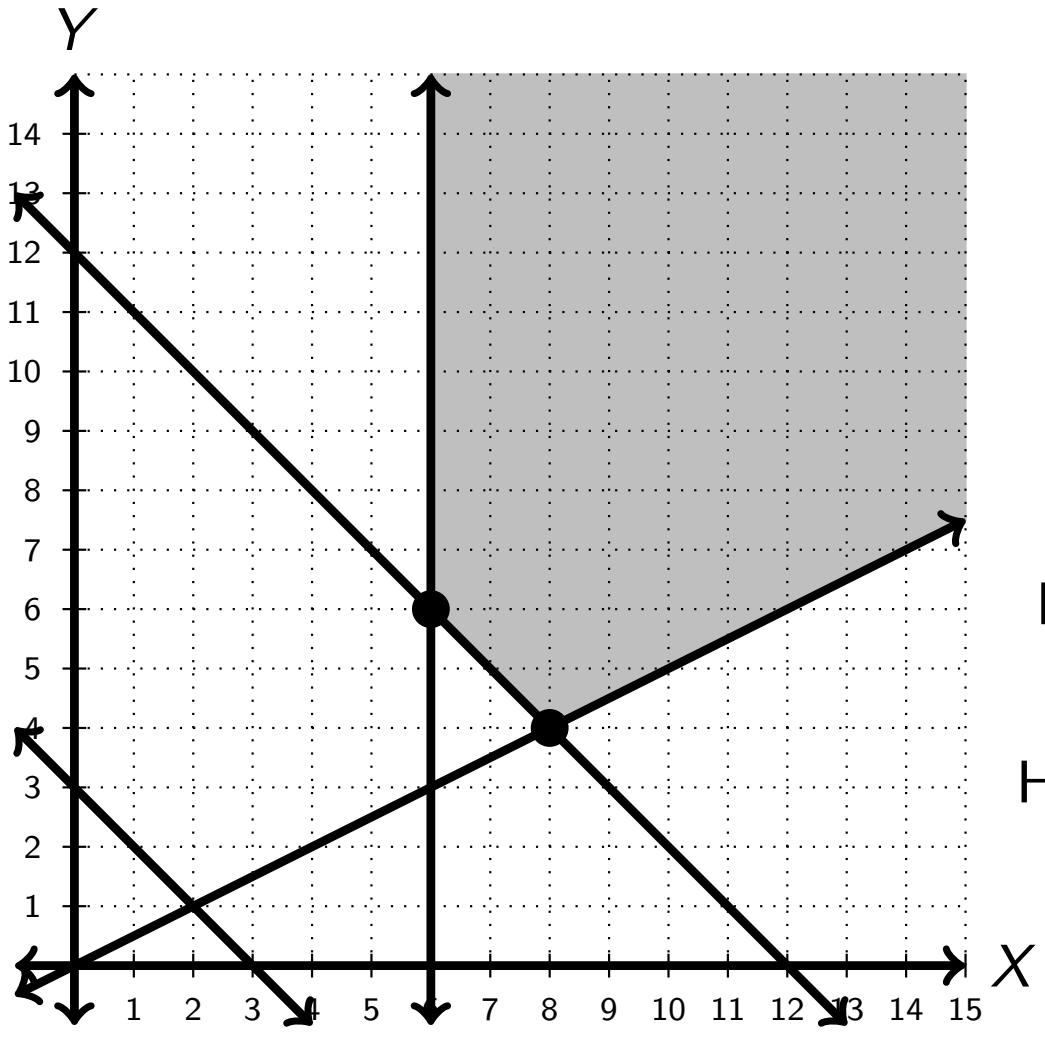
3.1: Where's the missing corner?



How many edges?

How many corners?

3.1: Where's the missing corner?



How many edges?

How many corners?

- This is called **unbounded** and it means we need to handle the "missing corner" specially.