DEPARTMENT OF MATHEMATICS

Ma 162 Second (practice) Exam March 4, 2013

Instructions: No cell phones or network-capable devices are allowed during the exam. You may use calculators, but you must show your work to receive credit. If your answer is not in the box or if you have no work to support your answer, you will receive no credit. The test has been carefully checked and its notation is consistent with the homework problems. No additional details will be provided during the exam.

Problem	**Maximum Score**	**Actual Score**
1	14	
2	14	
3	14	
4	14	
5	14	
6	15	
7	15	
Total	100	

NAME: _____ **Section:** _____

Last four digits of Student ID: _____

Show your work. Answers without justification receive no credit.

1. Matrix arithmetic. Do the following calculations if possible. If impossible, explain why.

(a) Add $\begin{bmatrix}1 & 2 & 3\end{bmatrix} + \begin{bmatrix}40 & 50 & 60\end{bmatrix}$

(b) Multiply $\begin{bmatrix}1 & 2 & 3\end{bmatrix} \times \begin{bmatrix}40 & 50 & 60\end{bmatrix}$

(c) Add $\begin{bmatrix}1 & 2 & 3\end{bmatrix} + \begin{bmatrix}40\\50\\60\end{bmatrix}$

(d) Multiply $\begin{bmatrix}1 & 2 & 3\end{bmatrix} \times \begin{bmatrix}40\\50\\60\end{bmatrix}$

(d') Multiply $\begin{bmatrix}40\\50\\60\end{bmatrix} \times \begin{bmatrix}1 & 2 & 3\end{bmatrix}$

(e) Find the inverse of $\begin{bmatrix}1 & 2 & 3\\0 & 1 & 4\\0 & 0 & 1\end{bmatrix}$.

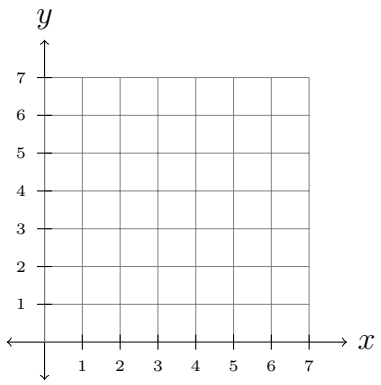
Show your work. Answers without justification receive no credit.

2. (a) Is $(x = 3, y = 1)$ on the correct side of $20x + 40y \leq 10$? Explain why or why not.

(b) Is $(x = 3, y = 1)$ a feasible solution to "maximize $P = 1.50x + 2.00y$ subject to $3x + 2y \leq 15$, $2x + 3y \leq 12$, $x \geq 0$, $y \geq 0$"?

(c) Is it optimal? Explain why or why not.

(d) What are the corners of the feasible region described by $3x + 2y \leq 15$, $2x + 3y \leq 12$, $x \geq 0$, $y \geq 0$? Make sure to show at least one full calculation.



Show your work. Answers without justification receive no credit.

3. Refer to this simplex tableau:

x	y	z	a	b	c	d	P	RHS
−5	0	3	0	2	1	0	0	33
4	1	2	0	3	0	0	0	21
3	0	1	1	4	0	0	0	10
2	0	0	0	5	0	1	0	44
1	0	−1	0	−6	0	0	1	55

(a) What is the basic solution indicated by this simplex tableau?

(b) Explain why it is feasible.

(c) Explain why it is not optimal.

(d) Which columns in this simplex tableau are eligible for pivoting?

(e) What happens if you pivot on a wrong column?

(f) Assuming we pivot the third column, which rows are eligible for pivoting?

(g) What happens if you pivot on a wrong row?

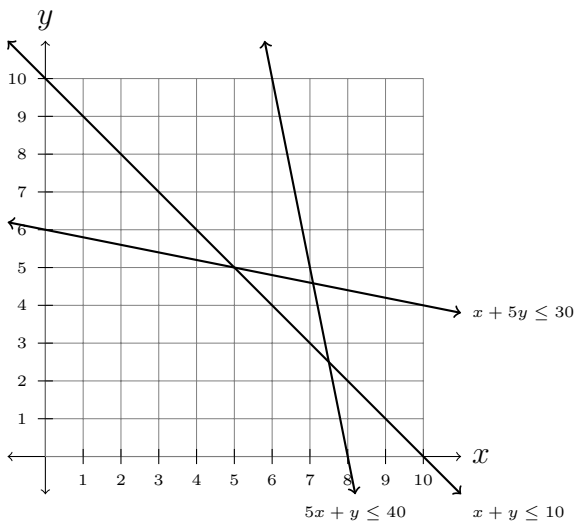
Show your work. Answers without justification receive no credit.

4. Do the row ops to pivot on the 3rd column, 3rd row, even if this is not the right row or column.

x	y	z	a	b	c	d	P	RHS
−5	0	3	0	2	1	0	0	33
4	1	2	0	3	0	0	0	21
3	0	1	1	4	0	0	0	10
2	0	0	0	5	0	1	0	44
1	0	−1	0	−6	0	0	1	55

Show your work. Answers without justification receive no credit.

5. Maximize $P = 1.50x + 2.00y$ subject to $5x + y \leq 40$, $x + y \leq 10$, $x + 5y \leq 30$, $x \geq 0$, $y \geq 0$ Make sure to (1) shade the region, (2) label the corners, (3) label where the maximum occurs and how big it is, and (4) why it must be the maximum.



(a) Sketch the shaded region.

(b) List the corners:

(c) Find the maximum (location and value):

(d) Explain why you have found the maximum:

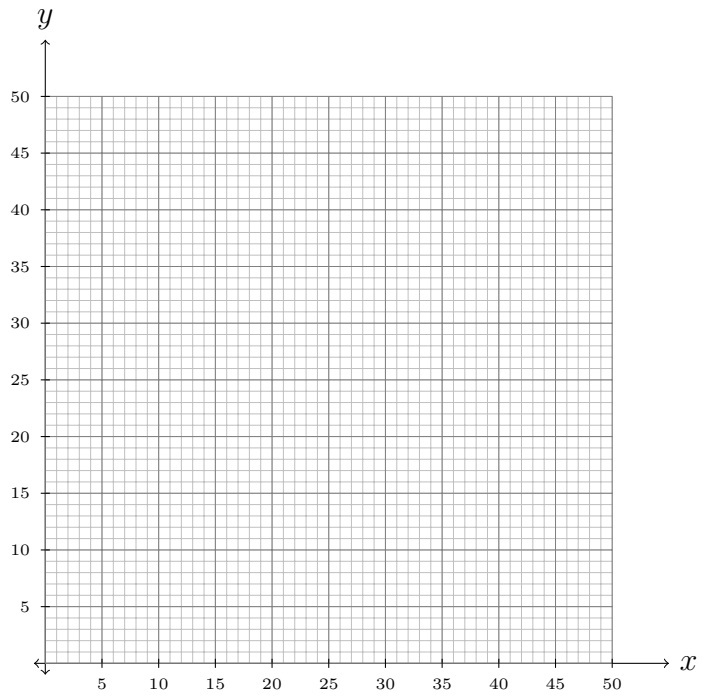
Show your work. Answers without justification receive no credit.

6. A company makes two products. The products require spending four resources that are limited by a budget. There is a minimum and maximum amount of each product that must and can be produced (some people have already pre-ordered each product, and there is a limited demand). Each product earns a certain amount of revenue.

	Usage				Demand		
	Resource 1	Resource 2	Resource 3	Resource 4	Minimum	Maximum	Revenue
Product 1	7	10	3	2	3	40	\$0.50
Product 2	7	5	6	2	9	40	\$1.05
Budget	350	405	255	500			

Give a recommendation to maximize revenue while using only the limited resources, meeting the minimum demand, and not producing more than the maximum demand:

Number of Product 1:	Bottom line Revenue:
Number of Product 2:	
Leftover Resource 1:	
Leftover Resource 2:	
Leftover Resource 3:	
Leftover Resource 4:	



Show your work. Answers without justification receive no credit.

7. A company makes 3 products using 5 limited resources. The resource usage and revenue for each product and the budget for each resource are given in the following table.

	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Revenue
Product 1	7	10	3	2	8	\$0.75
Product 2	7	5	6	2	5	\$1.05
Product 3	7	8	9	20	6	\$1.50
Budget	350	405	255	500	320	

How much of each product should the company make to maximize revenue while remaining under budget?

Give a plain English recommendation:

Now justify your answer with mathematics. You may find the following RREF calculation useful:

$$\left[\begin{array}{rrrrrrrrr|r} 7 & 7 & 7 & 1 & 0 & 0 & 0 & 0 & 0 & 350 \\ 10 & 5 & 8 & 0 & 1 & 0 & 0 & 0 & 0 & 405 \\ 3 & 6 & 9 & 0 & 0 & 1 & 0 & 0 & 0 & 255 \\ 2 & 2 & 20 & 0 & 0 & 0 & 1 & 0 & 0 & 500 \\ 8 & 5 & 6 & 0 & 0 & 0 & 0 & 1 & 0 & 320 \\ -0.75 & -1.05 & -1.50 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[\substack{R_4 - \frac{1}{3}R_3 \\ R_5 - \frac{5}{6}R_3 \\ R_6 + .175R_3}]{\substack{R_1 - \frac{7}{6}R_3 \\ R_2 - \frac{5}{6}R_3 \\ \frac{1}{6}R_3}} \left[\begin{array}{rrrrrrrrr|r} \frac{7}{2} & 0 & \frac{-7}{2} & 1 & 0 & \frac{-7}{6} & 0 & 0 & 0 & \frac{105}{2} \\ \frac{15}{2} & 0 & \frac{1}{2} & 0 & 1 & \frac{-5}{6} & 0 & 0 & 0 & \frac{385}{2} \\ \frac{1}{2} & 1 & \frac{3}{2} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{85}{2} \\ 1 & 0 & 17 & 0 & 0 & \frac{-1}{3} & 1 & 0 & 0 & 415 \\ \frac{11}{2} & 0 & \frac{-3}{2} & 0 & 0 & \frac{-5}{6} & 0 & 1 & 0 & \frac{215}{2} \\ \frac{-9}{40} & 0 & \frac{3}{40} & 0 & 0 & \frac{7}{40} & 0 & 0 & 1 & \frac{357}{8} \end{array}\right] \xrightarrow[\substack{R_4 - \frac{2}{7}R_1 \\ R_5 - \frac{11}{7}R_1 \\ R_6 + \frac{9}{140}R_1}]{\substack{\frac{2}{7}R_1 \\ R_2 - \frac{15}{7}R_1 \\ R_3 - \frac{1}{7}R_1}} \left[\begin{array}{rrrrrrrrr|r} 1 & 0 & -1 & \frac{2}{7} & 0 & \frac{-1}{3} & 0 & 0 & 0 & 15 \\ 0 & 0 & 8 & \frac{-15}{7} & 1 & \frac{5}{3} & 0 & 0 & 0 & 80 \\ 0 & 1 & 2 & \frac{-1}{7} & 0 & \frac{1}{3} & 0 & 0 & 0 & 35 \\ 0 & 0 & 18 & \frac{-2}{7} & 0 & 0 & 1 & 0 & 0 & 400 \\ 0 & 0 & 4 & \frac{-11}{7} & 0 & 1 & 0 & 1 & 0 & 25 \\ 0 & 0 & \frac{-3}{20} & \frac{9}{140} & 0 & \frac{1}{10} & 0 & 0 & 1 & 48 \end{array}\right]$$