MA162: Finite mathematics

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University of Kentucky

April 15th, 2013

Schedule:

- HW 1.1-1.4, 2.1-2.6, 3.1-3.3, 4.1, 5.1-5.3, 6A-6C, 7A (Late)
- HW 7B due Friday, Apr 19, 2013
- HW 7C due Friday, Apr 26, 2013
- Final Exam Tuesday, Apr 30, 2013 from 6pm to 8pm (new rooms on website)

Today we cover 7.2-7.3 ("chapter 6 works") through example problems

7.2: Just count for probability

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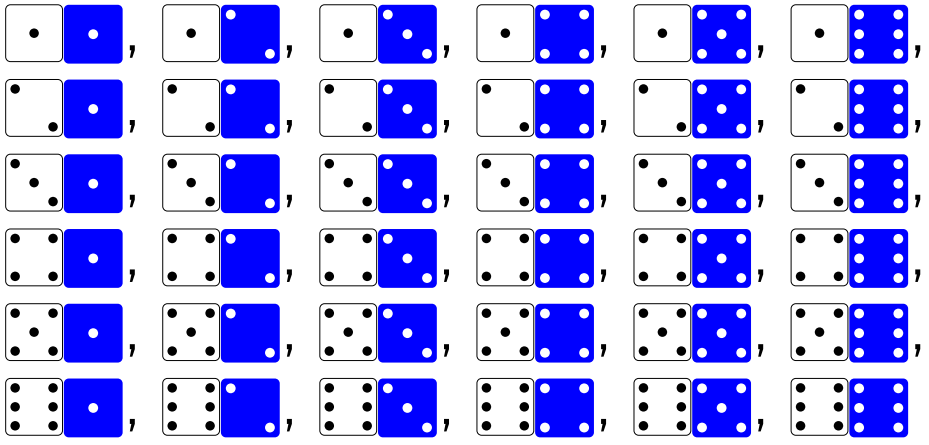

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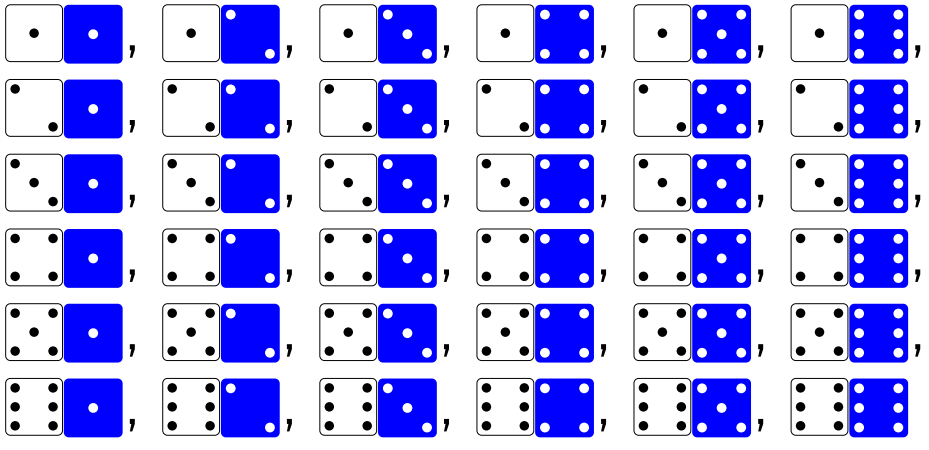

- The second row and the fifth column work: $P = \frac{6+6-1}{(6)(6)} = \frac{11}{36}$

7.2: Crazy counting

- A standard deck has 13 ranks (A,2,3,4,5,6,7,8,9,10,J,Q,K) and 4 suits ($\heartsuit, \diamondsuit, \clubsuit, \spadesuit$)
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- What is the probability of getting at least 2 aces out of 3 cards?
- Two ways to get at least 2 aces: exactly 2 or exactly 3.

$$P(\text{exactly 2}) = \frac{C(4 \text{ aces}, 2 \text{ in the hand})C(48 \text{ not aces}, 1 \text{ in the hand})}{C(52 \text{ cards}, 3 \text{ in the hand})} = \frac{\frac{(4)(3)}{(2)(1)}\frac{(48)}{(1)}}{\frac{(52)(51)(50)}{(3)(2)(1)}} = \frac{72}{5525}$$

$$P(\text{exactly 3}) = \frac{C(4,3)}{C(52,3)} = \frac{\frac{(4)(3)(2)}{(3)(2)(1)}}{\frac{(52)(51)(50)}{(3)(2)(1)}} = \frac{1}{5525}$$

$$P(\text{at least 2}) = \frac{C(4,2)C(48,1) + C(4,3)}{C(52,3)} = \frac{73}{5525} \approx 1.3\%$$

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- $P(E - F) = P(E) - P(E \cap F) = 40\% - 10\% = 30\%$

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- $Pr(E) = Pr(E \cap F) + Pr(E - F)$
- Every counting problem formula you can imagine has a probability counterpart

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$$\frac{91}{216} = 1 - \left(1 - \frac{1}{6}\right)^3$$

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