MA322-001 Jan 22 quiz Name: Key v2

1.2.1 (HW1.2#13) Describe the general solution to the system of equations represented by the following augmented matrix. Make sure your solution has no circular or nested definitions (free variables should be labelled as free, basic variables should be defined in terms of free variables and not in terms of other basic variables; look these words up in 1.2 if you don't know what they mean).

Basic Basic Free Basic Free

x_1	x_2	x_3	x_4	x_5	#
1	−3	0	−1	0	−2
0	1	0	0	−4	1
0	0	0	1	9	4
0	0	0	0	0	0

$\rightarrow x_1 = 3x_2 + 4x_4 - 2$

but x_2, x_4 are basic, not free, so not supposed to use them. Do Row Ops to fix

$$\begin{cases} x_1 = 5 + 3x_5 \\ x_2 = 1 + 4x_5 \\ x_3 \text{ is free} \\ x_4 = 4 - 9x_5 \\ x_5 \text{ is free} \end{cases}$$

new $R_1 = R_1 + 3R_2 + R_4$

new $R_1 = [1\ 0\ 0\ 0\ \ 3(-4)+9\ \ -2+3(1)+4]$

new $R_1 = [1\ 0\ 0\ 0\ \ -3\ \ 5\ ]$

1.3.1 Write $b = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$?

Just look at picture

$2v_1 + 2v_2 = b$

$2\begin{bmatrix} -1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

1.6.1 Balance this equation: $\underline{1}\ C_3H_8 + \underline{5}\ O_2 \rightarrow \underline{3}\ CO_2 + \underline{4}\ H_2O$

$$\begin{bmatrix} 3&0&-1&0 \\ 8&0&0&-2 \\ 0&2&-2&-1 \end{bmatrix} \xrightarrow{R_1=3R_1-R_2} \begin{bmatrix} 1&0&-3&2 \\ 8&0&0&-2 \\ 0&2&-2&-1 \end{bmatrix} \xrightarrow{R_2=R_2-8R_1} \begin{bmatrix} 1&0&-3&2 \\ 0&0&24&-18 \\ 0&2&-2&-1 \end{bmatrix} \xrightarrow[R_2/2,R_3/24]{R_3\leftrightarrow R_2} \begin{bmatrix} 1&0&-3&2 \\ 0&1&-1&-1/2 \\ 0&0&1&-3/4 \end{bmatrix} \xrightarrow[R_2=R_2+R_3]{R_1=R_1+3R_3} \begin{bmatrix} 1&0&0&-1/4 \\ 0&1&0&-5/4 \\ 0&0&1&-3/4 \end{bmatrix}$$

$x_1 = 0 + 1/4\ x_4$

$x_2 = 0 + 5/4\ x_4$

$x_3 = 0 + 3/4\ x_4$

Set $x_4 = 4$

$x_1 = 1$

$x_2 = 5$

$x_3 = 3$

$x_4 = 4$

You should get this back on Fri Jan 24. The following note is for you to read then:

Before next class (Mon Jan 27) (a) reread 1.4 and fix your notes, (b) read 1.5 and get your notes ready for next class, and (c) do HW1.4 #1,3,5*,7*,13,25 For 5 and 7 answer the question, but notice there is no calculation required. You may find that 5, 7, and 25 go together nicely.