MA322-001 Jan 24 quiz

Name: Key

1.3.1 (HW1.3#11/13) Determine if b is a linear combination of the columns of the matrix A. If it is, write out the linear combination; if not, explain why not.

$$A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

Solve $x_1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

new $R_2 = R_2 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

new $R_3 = R_3 + (-2) R_2$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so $x_1 = 2 - 5x_3$
$x_2 = 3 - 4x_3$
x_3 is free

Many Answers but

$$1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + 2.2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + 0.2 \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$$

works. So does

$$2 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + 0 \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$$

1.3.2 Write a vector equation that is equivalent to the following system of equations:

$$\left\{ \begin{array}{r} x_2 + 5x_3 = 0 \\ 4x_1 + 6x_2 - x_3 = 0 \\ -x_1 + 3x_2 - 8x_3 = 0 \end{array} \right\}$$

$$x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1.4.1 Write a matrix equation (of the form $Ax = b$ for x, b vectors and A a matrix) for that same system.

$$\underbrace{\begin{bmatrix} 0 & 1 & 5 \\ 4 & 6 & -1 \\ -1 & 3 & -8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

You should get this back on Mon Jan 27. The following note is for you to read then:

Before next class (Wed Jan 29) (a) reread 1.5 and fix your notes, (b) get your questions ready for next class, and (c) do HW1.5 #1, 3, 5, 7, 11, 13, 15, 19, 21.

MA322-001 Jan 24 worksheet

Stoichiometry!

(a) Balance this reaction: $\underbrace{\;1\;} C_3H_8 + \underbrace{\;5\;} O_2 \to \underbrace{\;3\;} CO_2 + \underbrace{\;4\;} HO_2$

See (f)

(b) Find scalars x_1, x_2, x_3, and x_4 (not all zero) so that

$$\underbrace{\;x_1\;} \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} + \underbrace{\;x_2\;} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \underbrace{\;x_3\;} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + \underbrace{\;x_4\;} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

See (f)

(c) Solve the equations

$$\begin{cases} 3x_1 = x_3 \\ 8x_2 = 2x_4 \\ 2x_2 = 2x_3 + x_4 \end{cases}$$

See (e)

(d) Row reduce this matrix:

$$\left[\begin{array}{cccc|c} 3 & 0 & -1 & -0 & 0 \\ 8 & 0 & -0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

Hint: $$\left[\begin{array}{cccc|c} 3 & 0 & -1 & -0 & 0 \\ 8 & 0 & -0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right] \xrightarrow[R_2-(8/3)R_1]{R_1/3,\, R_3/2} \left[\begin{array}{cccc|c} 1 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 8/3 & -2 & 0 \\ 0 & 1 & -1 & -1/2 & 0 \end{array}\right] \xrightarrow[R_2 \leftrightarrow R_3]{(3/8)R_2} \left[\begin{array}{cccc|c} 1 & 0 & -1/3 & 0 & 0 \\ 0 & 1 & -1 & -1/2 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{array}\right] \xrightarrow[R_2+R_3]{R_1+(1/3)R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/4 & 0 \\ 0 & 1 & 0 & -5/4 & 0 \\ 0 & 0 & 1 & -3/4 & 0 \end{array}\right]$$

This is right. :)

(e) What is the general solution?

$X_1 = \frac{1}{4} X_4$ $\quad$ $X_3 = \frac{3}{4} X_4$

$X_2 = \frac{5}{4} X_4$ $\quad$ X_4 is free

(f) What is a non-zero solution with all integers?

$X_4 = 4$ then $\quad X_1 = 1 \quad X_3 = 3$

$X_2 = 5 \quad X_4 = 4$