MA322-001 Feb 5 quiz   Name: Key

1.7.1 Solve $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ when $A = \begin{bmatrix} 1 & 5 & 7 \\ 2 & 5 & 4 \\ 3 & 5 & 1 \end{bmatrix}$, $\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $\vec{\mathbf{b}} = \begin{bmatrix} 14 \\ 8 \\ 2 \end{bmatrix}$ using the observation that $-3\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix}$.

$b = 2V_3 = -6v_1 + 4v_2$

You shouldn't need to row reduce.

$X_1 = -6 + 3X_3$

$X_2 = 4 - 2X_3$

X_3 is free

Exam: Is $\vec{\mathbf{x}} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$ on the plane containing $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}$?

(a) Write the parametric/vector equation $\vec{\mathbf{x}} = \ldots$ with $\vec{\mathbf{x}}$ actually having variables in it.

$$\vec{X} = \vec{V}_1 + s(\vec{V}_2 - \vec{V}_1) + t(\vec{V}_3 - \vec{V}_1)$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} + s\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(b) Write down how you would solve that equation, or why no one can solve it.

$X_1 = 5$, $X_2 = 6$, $X_3 = 7$

$$\begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} + s\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = s\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$, but $s=2$, $t=3$ gives $\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ not $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

no solution

1.7: Linear Dependence MA322-001 Feb 5 Worksheet

Why does $x_1 \begin{bmatrix}1\\1\\0\\0\end{bmatrix} + x_2 \begin{bmatrix}0\\0\\1\\5\end{bmatrix} + x_3 \begin{bmatrix}2\\2\\1\\5\end{bmatrix} = \vec{\mathbf{b}}$ either have no solutions or infinitely many solutions?

Be prepared to answer to the class. $V_3 = 2V_1 + V_2$. We can replace an X_3 with $2 \cdot X_1$'s and an X_2

$X_1 = ? - 2X_3$

$X_2 = ? - X_3$

X_3 is free

"No" versus "infinitely many" depends on the numbers in $\vec{\mathbf{b}}$ of course.
How do you tell which $\vec{\mathbf{b}}$ have any solutions?

Be prepared to answer to the class.

$$b = X_1 \begin{bmatrix}1\\1\\0\\0\end{bmatrix} + X_2 \begin{bmatrix}0\\0\\1\\5\end{bmatrix} + X_3 \begin{bmatrix}2\\2\\1\\5\end{bmatrix} = \begin{bmatrix}X_1 + 2X_3\\X_1 + 2X_3\\X_2 + X_3\\5X_2 + 5X_3\end{bmatrix}$$

so $b_1 = b_2$

$b_4 = 5b_3$

If $b_1 = b_2$ and $b_4 = 5b_3$, then ∞ solutions. Otherwise none.

Takeaway: If the columns of a matrix A are linearly independent, then there is at most one solution $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$, never infinitely many. The only way to get infinitely many is (the reason above; fill in the official version here).

Infinitely many solutions only when one of the columns is dependent / redundandant. $Ax = 0$ are exactly the ways in which columns are redundant.

Weird question: You should have asked: "hrm, what about the rows? do the rows need to be linearly independent? What's up with that?" So... what IS up with that?

Best answer wins $\approx$ \$5 gift certificate!