1.7: Linear Dependence MA322-001 Feb 5 Worksheet

Why does $x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 2 \\ 1 \\ 5 \end{bmatrix} = \vec{\mathbf{b}}$ either have no solutions or infinitely many solutions?

Be prepared to answer to the class.

"No" versus "infinitely many" depends on the numbers in $\vec{\mathbf{b}}$ of course.
How do you tell which $\vec{\mathbf{b}}$ have any solutions?
Be prepared to answer to the class.

Takeaway: If the columns of a matrix A are linearly independent, then there is at most one solution $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$, never infinitely many. The only way to get infinitely many is (the reason above; fill in the official version here).

Weird question: You should have asked: "hrm, what about the rows? do the rows need to be linearly independent? What's up with that?" So... what IS up with that?

MA322-001 Feb 5 quiz Name:________________________

1.7.1 Solve $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ when $A = \begin{bmatrix} 1 & 5 & 7 \\ 2 & 5 & 4 \\ 3 & 5 & 1 \end{bmatrix}$, $\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $\vec{\mathbf{b}} = \begin{bmatrix} 14 \\ 8 \\ 2 \end{bmatrix}$ using the observation that $-3\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix}$.

You shouldn't need to row reduce.

Exam: Is $\vec{\mathbf{x}} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$ on the plane containing $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}$?

(a) Write the parametric/vector equation $\vec{\mathbf{x}} = \dots$ with $\vec{\mathbf{x}}$ actually having variables in it.

(b) Write down how you would solve that equation, or why no one can solve it.