MA322-001 Feb 7 Quiz Name: Key

1.7.1 In each matrix decide if the columns are linearly independent. If they are not, cross out the **minimum** number of columns to make them linearly independent.

$\begin{bmatrix}1&2\\4&5\\6&7\end{bmatrix}$ $\begin{bmatrix}1&2&\cancel{3}\\4&5&\cancel{9}\\6&7&\cancel{13}\end{bmatrix}$ $\begin{bmatrix}\cancel{0}&1&2\\\cancel{0}&3&4\\\cancel{0}&5&6\end{bmatrix}$ $\begin{bmatrix}1&2\\1&1\\0&0\end{bmatrix}$ $\begin{bmatrix}1&\cancel{1}\\1&\cancel{1}\\0&\cancel{0}\end{bmatrix}$

Ind

$$\begin{matrix}1&&2&&3\\4&+&5&=&9\\6&&7&&13\end{matrix}$$

X_3 is free

$X_1 = ? - X_3$

$X_2 = ? - X_3$

X_1 is free

Ind

X_2 is free

$x_1 = ? - X_2$

1.8.1 Write down a matrix that sends $\vec{x} = \begin{bmatrix}1\\0\\0\end{bmatrix}$ to $\vec{b} = \begin{bmatrix}2\\3\end{bmatrix}$, but also sends $\vec{x} = \begin{bmatrix}0\\1\\0\end{bmatrix}$ to $\vec{b} = \begin{bmatrix}4\\5\end{bmatrix}$ and $\vec{x} = \begin{bmatrix}0\\0\\1\end{bmatrix}$ to $\vec{b} = \begin{bmatrix}6\\7\end{bmatrix}$.

$$X_1\begin{bmatrix}\ \\ \ \end{bmatrix} + X_2\begin{bmatrix}\ \\ \ \end{bmatrix} + X_3\begin{bmatrix}\ \\ \ \end{bmatrix} = \vec{b}$$

$\begin{bmatrix}1\\0\\0\end{bmatrix}$: $1\begin{bmatrix}a_1\\a_2\end{bmatrix} + 0\begin{bmatrix}a_3\\a_4\end{bmatrix} + 0\begin{bmatrix}a_5\\a_6\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$ so $a_1 = 2$, $a_2 = 3$

$\begin{bmatrix}0\\1\\0\end{bmatrix}$: $0\begin{bmatrix}a_1\\a_2\end{bmatrix} + 1\begin{bmatrix}a_3\\a_4\end{bmatrix} + 0\begin{bmatrix}a_5\\a_6\end{bmatrix} = \begin{bmatrix}4\\5\end{bmatrix}$ so $a_3 = 4$, $a_4 = 5$

$\begin{bmatrix}0\\0\\1\end{bmatrix}$: $0\begin{bmatrix}a_1\\a_2\end{bmatrix} + 0\begin{bmatrix}a_3\\a_4\end{bmatrix} + 1\begin{bmatrix}a_5\\a_6\end{bmatrix} = \begin{bmatrix}6\\7\end{bmatrix}$ so $a_5 = 6$, $a_6 = 7$

$$A = \begin{bmatrix}a_1&a_3&a_5\\a_2&a_4&a_6\end{bmatrix} = \begin{bmatrix}2&4&6\\3&5&7\end{bmatrix}$$

1.8: Linear transformations MA322-001 Feb 7 Worksheet

Usually we thought of $\vec{b}$ as given, and tried to solve for $\vec{x}$. We've started to let $\vec{b}$ vary, and asked if we can even find a $\vec{x}$. That led us to just try an $\vec{x}$, to see which $\vec{b}$ we got. In other words, we can view a matrix like A as a function that that takes $\vec{x}$ and gives us $\vec{b} = A\vec{x}$ as the answer.

This function $A(\vec{x}) = A\vec{x}$ is better than the ordinary function. It is **linear**. $A(\vec{x}+\vec{y}) = A(\vec{x}) + A(\vec{y})$ and $A(c\vec{x}) = cA(\vec{x})$.

Consider $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$. What does it do to a vector $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$?

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 + 0 \\ 0 + x_2 \\ x_1 + x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_1 + x_2 \end{bmatrix}$$

(This matrix is known as the "checksum" in coding theory.)

Does $A(\vec{x}) = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ have a solution?

$$A(\vec{x}) = \begin{bmatrix} x_1 \\ x_2 \\ x_1 + x_2 \end{bmatrix} \quad \text{so solve} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_1 + x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$x_1 = 3$, $x_2 = 4$, but $x_1 + x_2 = 7 \neq 5$

No solution.

Takeaway: $\vec{x} \xrightarrow{A} \vec{b}$

Weird question: If $\vec{x}$ is small, is $\vec{b}$ small? There is a number $\|A\|_2$ so that if x is small $Ax = b$ is no more than $\|A\|_2$ times as big as x.

($\sqrt{3}$ for the one up there!)